Research and Development and Competition

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Abstract

This paper studies the relationship between firms' incentives to carry out R&D and competition in a stochastic continuous time setting. We show that the relationship depends on how competition intensity affects the expected profits of the innovation evaluated at the optimal stopping time relative to the current profits. We also show that when the firm has multiple ideas that can be researched sequentially, the probability of implementing the first idea falls as the correlation between the Brownian motions rises. Finally, when there are two firms with the ability to innovate, and firms innovate sequentially, firms are less prone to implement their ideas when XX **Keywords:** Bayesian Learning, Optimal Stopping, R&D, Competition Intensity.

JEL-Classification: G32, J24, L26, M13

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1 Introduction

Schumpeter (1942) argue that technological innovation often creates temporary monopolies, allowing abnormal profits that would soon be competed away by rivals and imitators. These temporary monopolies are necessary to provide the incentive for firms to develop new products and processes (see, also, Loury (1979), Grossman and Helpman (1991), Aghion and Howitt (1992), Caballero and Jaffe (1993), Martin (1993)). In contrast, Arrow (1962) shows that a monopoly facing no competition for existing and new technologies has fewer incentives to invest in process innovation than a firm in a perfectly competitive industry due to the replacement effect (see, also, Porter (1990), and Baily et al. (1995)).[1] Namely, a firm with a monopoly position in a market has a flow of profit that it enjoys if no innovation occurs. A monopolist can increase its profit by innovating but loses the profits from its old technology. While under competition, there is a significant return to innovation since under the old technology, firms make small or zero profits. After many years of scrutiny, the main conclusion of the literature at both the empirical and the theoretical level is that the relationship between R&D and competition intensity is sometimes positive, sometimes negative, and sometimes positive at a low competition intensity level and then negative at a high competition level.

This paper studies the relationship between R&D and competition. We model R&D as a sequential information acquisition problem à-la-Wald (1945). When the firm carries R&D, it receives a signal at each time, and beliefs are updated according to Baye's rule. We apply the methodology outlined in Araman and Caldentey (2022) to interpret the continuous-time beliefs as the stochastic limit (in weak convergence sense) of a pure-jump Bayesian learning process in which updates occur at discrete time epochs. A key consequence is that beliefs evolve according to a stochastic differential equation and are a martingale. Thus, the stopping time consists of a hitting time; that is, the first time when the stochastic belief-updating process hits a given subset of the state space, a decision is made. To solve the optimal stopping problem, we use the verification theorem in Araman and Caldentey (2022), which is based on Ito's Lemma and Dynkin's formula.

We consider an industry with two firms indexed with the same instantaneous discount factor that

[1] There is a literature inspired by the seminal work of Hart (1983), focusing on managerial incentives, such as Schmidt (1997), Aghion et al. (1997) and Aghion et al. (1999), that provides rationales for a positive correlation between competition and managerial effort that can be associated to innovation. This literature hinges on an unusual assumption that managers minimize innovation costs subject to the constraint that the firm does not go bankrupt, instead of the more common assumption of profit maximization.

engage in the following stochastic game in continuous time: initially, firm 1 chooses between implementing a non-disruptive innovation whose characteristics in terms of profitability–either good or bad– are initially unknown, discarding the innovation, or to conduct R&D for an optimally chosen time before making the decision. The bad idea is less profitable than the good idea. The bad idea is one whose implementation is more likely to fail than the good idea.

Each firm has three options: (i) to implement the idea immediately, (ii) to discard it immediately and keep producing with the current technology, or (iii) to engage in R&D to gather information about the profitability of the idea before deciding whether to implement it or discard it. A firm's decision to implement an idea or not do so is irrevocable.

We assume that firms' prior belief about the idea being bad–common to every firm– is common knowledge. Doing R&D has a constant marginal cost per unit of time. The marginal cost is no longer paid once a firm implements or discards its idea.

Following Athey and Schmutzler (2001), Boone (2000), and Schmutzler (2010), we assume that firms are endowed with a reduced-form profits function depending on the innovation profile and a parameter that measures competition intensity, such as the degree of product differentiation or the number of competitors. The reduced-form profits result from an unmodeled product-market game in which firms choose strategic variables such as prices or quantities. The market structure is exogenous and held constant.

First, we consider the case in which only one firm has an idea and the case in which the firm has more than one idea that must be investigated sequentially. Secondly, we consider the case in which both firms have ideas, which are also investigated sequentially.

Given that the discounted payment when the idea is good is larger than when it is bad, the firm's optimal strategy is characterized by the solution to a stochastic differential equation (SDE) arising from the agent's optimal stopping problem. The solution is a hitting time with a low and high threshold. When the posterior belief hits the low threshold, the firm stops doing R&D and implements the idea the project, and when it hits the high threshold, the firm stops R&D and discards the idea, in which case it gets the same profits that it got while doing R&D but it save its cost. We call this the regular profits.

The high threshold occurs when the solution to the SDE hits and is tangent to the firm's regular profits. The low threshold occurs where the solution is tangent to the locus of points that yields the expected discounted profits. The smooth-pasting and value-matching conditions when the idea is either executed or dropped are optimal conditions explained by the assumption that the firm can stop doing R&D and decide what to do at any time it wishes.

The R&D intensity, as measured by the difference between the two thresholds and the expected time, and the probability that the idea is implemented fall as regular profits increase when the profit from the innovation is held constant. Thus, the model confirms the existence of Arrow's replacement effect. However, when competition intensity is captured by a parameter such as the differentiation or number of firms, an increase in competition intensity not only decreases regular profits but also the profits from the idea. In this case, the relationship between R&D and competition is negative when the expected profits elasticity of competition is larger in absolute value than the regular profits elasticity of competition intensity. If the opposite happens, the relation becomes positive.

The rest of the paper is as follows. The next section discusses the related literature. Following this, in Section 3, we present the model. In Section **??**, we solve for the socially optimal acceptance/rejection/learning strategy. Then, we study the agent's problem. In Section **??**, the principal's optimal contract is derived. After this, Section **??** discusses two extensions regarding moral hazard and more general contracts. In the final section, we present concluding remarks and discuss future work.

2 Literature Review

This paper, first and foremost, contributes to the literature on R&D with asymmetric information and Bayesian Learning.

In our setting, each firm is faced with an optimal stopping problem with sequential information acquisition à-la-Wald (1945) (see also Wald and Wolfowitz, 1948). Rather than assuming that learning occurs at discrete time epochs, as it is customary in the sequential learning literature (see, e.g., Chernoff, 1959, 1972, Siegmund, 1985 and references therein), we adopt a continuous-time Bayesian formulation in which the agent's beliefs evolve as a martingale diffusion process. We apply the methodology outlined in Araman and Caldentey (2022) to interpret the agent's continuous-time beliefs as the stochastic limit (in weak convergence sense) of a pure-jump Bayesian learning process in which the agent updates his beliefs at discrete time epochs.[2]

[2]Continuous-time formulations of multi-armed bandit problems have received limited attention in the literature. Two

Numerous articles, including those by Roberts and Weitzman (1981), Moscarini and Smith (2001), Branco et al. (2012), Fudenberg et al. (2018), and Lang (2019), have extensively explored drift-diffusion models (DDMs), wherein signals follows a Brownian motion dynamics with unknown drift. Ulu and Smith (2009) studies the broadest setting of this problem, incorporating general probability distributions for payoffs and the decision maker's beliefs. They demonstrate that if the signal-generating process satisfies the monotone likelihood ratio property and the DM is risk neutral, the value functions and learning policies will satisfy natural monotonicity properties. Subsequently, Smith and Ulu (2017) extends the analysis to consider risk-averse decision-makers.

The study of the relationship between competition and innovation dates back to Schumpeter (1942), who argues that monopolies provides stronger incentives to innovate.[3] In contrast, there is a large literature that claims that an increase in competition intensity increases industry-wide innovation (see, for instance, Arrow (1962), Porter (1990), and Baily et al. (1995)) and, as a consequence of this, efficiency. However, both argue that to the extent that innovation cannot be kept fully private, the returns to the investment in it cannot be fully appropriated by the firm undertaking the investment and, therefore, firms will be reluctant to invest, leading to the underprovision of innovation in the economy. Furthermore, in a Schumpeterian economy there is a price to pay for rapid technological progress given by a market structure involving large firms with considerable market power. These views spurred a large theoretical and empirical literature studying the relationship between competitive pressure and innovation.

Aghion and Bolton (1997), Aghion et al. (2001) and Aghion et al. (2005) show that there is an inverted U-shaped relationship between competition intensity and industry-wide innovation. This stems from the fact that innovations are strategic substitutes and there are large spillovers, defined as the probability that a follower moves one step ahead of the leader without investing in innovation. Their model assumes that the lagging firm is always one step behind by assuming that when the leader innovates by one step (which is the maximum allowed), the follower copies, at no cost, the technology the leader had before the new innovation is discovered. Vives (2008), López and Vives (2016) and

notable exceptions include the recent papers by Wager and Xu (2023) and Fan and Glynn (2021) in the context of multi-armed bandit problems who study a similar type of asymptotic regime and diffusion limits as the ones considered in this paper.

[3]This relationship has been considered key to understand growth and development (see, e.g., Aghion et al. (2001), market concentration (see, e.g., Athey and Schmutzler (2001)), and technology adoption (see, e.g., Reinganum (1989)) among many other important economic issues.

Gilbert et al. (2018) also argue in favor of an "inverted-U" relationship between competition and innovation.[4] Marshall and Parra (2019) study a dynamic innovation model and provide conditions regarding the relationship between the profit gap and the intensity of competition for when the latter increases or decreases industry innovation and welfare. Letina (2016) shows that an increase in the intensity of competition, defined as any exogenous change which decreases firm profits, increases the variety of approaches to innovation, and decreases the amount of R&D duplication in equilibrium. Thus, the total amount invested in R&D may either increase or decrease with competition.

Schmutzler (2013) studies a reduced-form profit duopoly and defines that an increase in a parameter means a higher competition intensity when it decreases price-cost margins, increases the impact of own innovation, and decreases the impact of competitors' innovation on its own demand, demand increases with its own innovation, and price-cost margins fall with it. He shows that if the well-known Hahn's stability condition holds, the firm i's investment is weakly increasing and firm j's weakly decreasing with competition intensity only if firm i's marginal return rises with competition intensity, firm j's marginal return falls with it, and firms' investments are substitutes. We generalize Schmutzler's (2013) results by considering more than two firms, innovations that could be either complements or substitutes, and more general conditions regarding the impact of competition intensity in marginal returns. Furthermore, the conditions for increasing industry-wide innovation in a duopoly cannot be generalized to oligopolies.

Boone (2000), also using a reduced-form model, analyzes the effects of competitive pressure on firms' incentives to innovate. He considers both product and process innovations. The effects of a rise in competitive pressure on a firm's incentives to invest in these innovations depend on its efficiency level relative to that of its opponents. He provides conditions under which a rise in competitive pressure increases each firm's innovation in process innovations and shows that a rise in competitive pressure cannot raise both product and process innovations at the industry level. Boone (2001) considers the effects of the intensity of product market competition on R&D incentives. He proposes four axioms that a measure of competition intensity should satisfy and shows that the axioms imply the existence of different types of non-monotone relations between the intensity of competition and the value of

[4] Gilbert et al. (2018), using Aghion et al.'s (2005) model, show that when $n = 2$, innovation is decreasing in competition intensity–product differentiation– when large externalities are not allowed, and when there are $n > 2$ firms, innovation increases with the number of rivals. They also show that an inverted U-shape relationship may arise without externalities when there are more than two firms, leaders' profits decrease in the number of leaders, followers always make less money than leaders, and innovations are substitutes.

innovation. He requires the same sufficient conditions that we do for industry-wide innovation to increase, but he also needs symmetry.[5]

The empirical evidence is consistent with the fact that, theoretically, the prediction regarding the relationship between competition and innovation is ambiguous. Baily et al. (1995), Blundell et al. (1995), Nickell (1996) provide support for a positive relationship between industry-wide innovation and competition, while Aghion et al. (2005) provide evidence in favor of an "inverted-U" relationship between competition and industry-wide innovation. Hashmi (2013) finds a negative and robust relationship between competition and industry-wide innovation. To reconcile the mildly negative relationship in the U.S. data with the inverted-U relationship found by Aghion et al. (2005) in the U.K. data, he tests whether U.K. manufacturing industries are technologically more neck-and-neck than their counterparts in the United States and finds support for this.

Beneito et al. (2015), using panel data of Spanish manufacturing firms for 1990-2006, find that greater product substitutability and higher costs of entry lead to more process innovation, but less product innovation, whereas increases in market size increase both product and process innovation. Kretschmer et al. (2012) find that higher competitive pressure, as measured by the elimination of exclusive territories in the French automotive market, lowers process innovation and increases product innovation. They pay special attention to the impact of the scale of operations and innovation strategies by considering multiple innovations. This highlights the importance of the output or scale effect on innovation. Goettler and Gordon (2011) study competition between AMD and Intel by assuming as counterfactual that Intel is a monopoly. They find competition from AMD had a negative impact on the speed of innovation, but overall it has had a positive effect on consumer welfare because the competition effect on prices has offset the lower quality. They also consider the counterfactual scenario of a symmetric duopoly where the two firms have the same demand brand fixed effects and innovation intensity parameters. They report that investment in R&D, innovation rates, and average quality decline. However, welfare increases by \$34 million (1.2%), industry profits decline by \$8 million, and social surplus increases by \$26 million (less than 1%). This stems from the fact that prices decline and this effect more than offsets the quality decline.

Hashmi and Biesebroeck (2016) study the effect of market power on innovation in the automobile industry. Their main finding shows that adding another firm would lower the rate of innovation in

[5] Earlier work on stochastic patent races (e.g., as surveyed by Reinganum (1989)) considers the relationship between the number of competitors and the rate of innovation with mixed results.

this industry, and this effect is magnified with higher-quality entrants, but industry-wide innovation increases in most market structures. They also find that holding their own quality constant, innovation is declining in average rival quality but increasing in quality dispersion. Hence, the impact of competition at the individual firm level is negative, but at the industry-wide level is rising because of entry.

Igami (2017) studies the relationship between competition and innovation by focusing on the propensity to innovate of new entrants relative to incumbents in the hard drive industry. He finds that despite strong preemptive motives and a substantial cost advantage over entrants, cannibalization makes incumbents reluctant to innovate, which can explain at least 57% of the incumbent-entrant innovation gap. Hence, the replacement effect seems stronger than the preemption effect. Igami and Uetake (2019) study a stochastically alternating-move game of dynamic oligopoly and estimate it using data from the hard disk drive industry, in which a dozen global players consolidated into only three in the last 20 years. They find plateau-shaped equilibrium relationships between competition and innovation, with heterogeneity across time and productivity.

Lampe and Moser (2013) find that patent pools in the 19th-century sewing machine industry decreased the patenting intensity of pool members, and Lampe and Moser (2016) find that patent pools decreased patenting intensity and citations across 20 industries. The underlying mechanism behind this relationship is that patent pools weaken competition in R&D, decreasing innovation output.

Hence, the empirical and theoretical relationship between innovation and competition is a-priori ambiguous and depends on the measure of competition used and the type of innovation studied. Furthermore, this conclusion ignores the role that financing constraints have on firms' investment in innovation. Indeed, Arrow's (1962) argue, foreshadowed by Schumpeter, that a key determinant of innovation is the existence of a wedge between the rate of return required by an entrepreneur investing his own funds and that required by external investors due to incomplete pledgeability of innovation outcomes, and the fact that security markets are incomplete and unable to ensure investors of the high uncertainty involved in innovative activities due to the trial and error nature of them. Thus, unless an inventor is already wealthy, innovation activity is bound to be subject to credit constrained.[6]

While we share with these papers the notion of incentive for experimentation in Bayesian environments and optimal stopping in continuous time, none of the papers study a setting like ours where the main

[6]For reviews of the empirical literature regarding the relationship between innovation and credit access see, for instance, Kerr and Nanda (2015) and Hall and Lerner (2010)).

drivers of inefficiencies are the fact that the choice between the risky and safe arm bandit is fully delegated to the agent and he can quit any time before committing to the risky arm, learning is private, and it is impossible to contract on certain states and the history of experimentation. Our setting captures, albeit in a simple form, many real-life situations that these other papers cannot speak to.

3 Model Setup

Let's consider an industry with n firms, indexed, $i \in \mathcal{I} \equiv \{1, \ldots, n\}$, with the same instantaneous discount factor r that engage in the following stochastic game in continuous time: at $t = 0$, firms choose whether or not to implement an innovation whose profitability is initially unknown for everyone. Each firm has three options: (i) implement the project immediately, (ii) reject it immediately and perform with the current technology, or (iii) spend time doing R&D to gather information about the profitability of the innovation before deciding whether to implement it or to reject it and take the outside option. The firm's decision to implement or not do so is irrevocable.

The idean can be either "bad" or "good" to implement. We will use θ to represent the unknown type of innovation, with $\theta = B$ indicating that it is bad to implement and $\theta = G$ indicating that it is good to implement. The prior belief –common to every firm– that the innovation is hard to implement is denoted by $\delta^i = \mathbb{P}(\theta^i = D)$. We will assume that $\delta^i \in (0, 1)$, innovations are distributed i.i.d. across firms, and all firms have the same prior δ.

Let $d^{-i} \equiv (\ldots, d^{i-1}, d^{i+1}, \ldots) \in \{0, 1\}^n$ and $d \equiv (d^i, d^{-i})$. Firm i's profits when the idea is discarded are $\pi^i(0, d^{-i}; \mu)$ and those during the periods the firm does R&D are $\pi^i(0, d^{-i}; \mu) - c$, where c is the per-unit of time cost of R&D and μ is a parameter measuring competition intensity such as the product differentiation, the number of firms, etc.

When the firm implements the idea, and this is good to implement, firm i's profits are $\pi^{iG}(1, d^{-i}; \mu)$ and when the implementation is hard, profits are $\pi^{iB}(1, d^{-i}; \mu)$.

The profit function must be understood as the equilibrium profits of a sub-game that occurs after firms observe (d, μ), where firms compete in the product market by simultaneously choosing prices or quantities or any other strategic variable.[7] For this interpretation to be valid, we need to assume

[7]See, Athey and Schmutzler (2001), Schmutzler (2013) and Boone (2000, 2001) for a similar reduced-form approach.

that there is a Nash equilibrium selection in the product-market game that can be identified for every innovation profile d.[8] We assume that profits are weakly decreasing in μ, capturing that more intense product market competition decreases the profits of all firms in every possible state.

Assumption 1. *For all $i \in \mathcal{I}$,*

i) For all $d \in \{0,1\}^n$, $\pi^{iG}(1, d^{-i}; \mu) > \pi^{iB}(1, d^{-i}; \mu)$.

ii) For all $d^{-i} \in \{0,1\}^{n-1}$, $\pi^{i}(0, d^{-i}; \mu) - c > 0$.

iii) For all $d^{-j} \in \{0,1\}^{n-1}$ and $j \neq i$, $\pi^{i\theta}(1, d^{-j}; \mu) \gtrless \pi^{i\theta}(0, d^{-j}; \mu)$ and $\pi^{i}(1, d^{-j}; \mu) \gtrless \pi^{i}(0, d^{-j}; \mu)$.

Part 1 says that the profitability of the idea, ceteris paribus competitors' innovation decisions, is higher when its implementation is good than when it is hard. Part 2 establishes that firm i's profits are positive while doing R&D. Part 3 establishes that firm i's profits may either increase or decrease when a competitor implements its idea.

From here onwards, we will say that firm i has positive externalities on firm j when an increase its innovation (weakly) rises, ceteris paribus, firm i's profits and has negative externalities when the opposite happens.[9] For any pair of investments i and j, externalities could be either positive or negative.

Externalities could arise from technology spillovers, knowledge sharing, and/or incomplete appropriability, which may increase/decrease the productivity of other firms operating in similar technology areas, or strategic spillovers, reflecting product-market interactions that create an indirect link between the investment decisions of firms through their anticipated impact on product market competition.

A firm's strategy is given by a tuple (τ^i, d^i), where $\tau^i \geq 0$ is the (possibly random) time firm i spends conducting R&D, and $d^i \in \{0,1\}$ is the firm's decision to either implementing the innovation $(d^i = 1)$ or discarding it $(d^i = 0)$.

[8] The standard approach is to assume that there is a unique locally stable equilibrium profile in the third stage that depends smoothly on investment and parameters. For instance, Milgrom and Roberts (1990) show that this holds for Bertrand's competition with differentiated goods, and Amir (1996) shows that this is the case for the Cournot game with fixed marginal costs when the inverse of the demand function is log-concave. To get uniqueness, it is usually assumed that product-market payoffs, given d, satisfy the well-known dominant diagonal condition.

[9] Externalities have been discussed in works such as Bloom et al. (2013), López and Vives (2016) and d'Aspremont and Jacquemin (1988).

For future reference, we let $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$ denote the usual filtration generated by B_t and by $\mathbb{T}$ the set of stopping times regarding $\mathbb{F}$. We will naturally require $\tau \in \mathbb{T}$ and $d \in \mathcal{F}_\tau$. Let $\delta_t = \mathbb{P}(\theta = 1 \mid \mathcal{F}_t)$ denote the agent's belief after conducting due diligence for t time units, where $\mathcal{F}_t$ is the collection of information the agent has gathered during this period. Henceforth, we denote by τ the amount of time the agent spends on due diligence activities.

A firm's information acquisition process is such that its belief δ_t, after conducting R&D for t units of time, evolves over this period according to the stochastic differential equation (SDE)

$$\mathrm{d}\delta_t = \delta_t \,(1 - \delta_t)\, \sigma \,\mathrm{d}B_t \qquad \text{with initial condition } \delta_0 = \delta, \tag{1}$$

where B_t is a Brownian motion and σ is a parameter that captures the firm's "speed" of learning.

The term of $\delta_t\,(1 - \delta_t)$ captures the intuitive idea that new information has less impact on posterior beliefs when there is greater certainty about the innovation's type—that is when δ_t is close to zero or one.

The continuous-time stochastic evolution of δ_t in (1) can be understood as the limit (in the sense of weak convergence) of a discrete-time belief process. In this discrete-time setting, every $\Delta > 0$ time unit, the agent gets a signal that is used to update the belief about the profitability of the innovation. Let δ_μ^Δ denote the agent's belief after collecting the n^{th} signal. Then, δ_n^Δ evolves according to Bayes's rule

$$\delta_n^\Delta = \delta_{n-1}^\Delta + (1 - \delta_{n-1}^\Delta)\, \delta_{n-1}^\Delta \left(\frac{1 - \mathcal{L}_n^\Delta}{\delta_{n-1}^\Delta + (1 - \delta_{n-1}^\Delta)\, \mathcal{L}_n^\Delta} \right),$$

where $\mathcal{L}_n^\Delta$ is the (random) likelihood ratio associated with the n^{th} signal. By letting $\Delta \downarrow 0$ and allowing $\mathcal{L}_n^\Delta$ converge to 1 (a.s.) at a rate of $O(\sqrt{\Delta})$, one can show that δ_n^Δ converges weakly to the continuous-time process δ_t in (1) (see Araman and Caldentey, 2022 for details). With this interpretation, we consider the continuous-time model to be a mathematically convenient approximation of a discrete-time model. The advantage of a continuous-time formulation is that it will enable us to apply the tools of stochastic calculus to the martingale process δ_t.

An assumption in th3e4 model is that the number of firms is exogenous. Thus, we are silent about the determinants of market structure (e.g., entry costs) and factors that could trigger changes in the number of competitors (e.g., mergers). Because the main purpose of our analysis is to understand how competition in both the product market and innovation impact innovation, independent of what factors could explain a change in competition, we chose not to endogenize market structure in any way.

4 The One Investor Case: The Optimal R&D Strategy

4.1 One Idea

In this section, we derive firm i's optimal strategy when all other firms do not have an idea to implement, and the current technology profile is given by d^{-i}.

The firm's optimal strategy solves:

$$\sup_{(\tau^i,d^i)\in\mathbb{T}\times\{0,1\}}\left\{\int_{\tau^i}^{\infty} e^{-r\,t}\left(d^i\mathbb{E}_\delta\pi^{i\theta}(1,d^{-i};\mu)+(1-d^i)\pi^i(0,d^{-i},\mu)\right)dt+\int_0^{\tau^i} e^{-r\,t}(\pi^i(0,d^{-i},\mu)-c)dt\right\} \tag{2}$$

subject to

$$\mathrm{d}\delta_t=\delta_t\,(1-\delta)\,\sigma\,\mathrm{d}B_t \quad \text{and} \quad \delta_0=\delta,$$

where $r>0$ is the agent's discount factor and $\mathbb{E}_\delta[\cdot]$ is the conditional expectation operator given a belief δ.[10]

To understand the equation above, notice that if the firm implements the idea at time τ, it obtains an expected flow payoff of $\mathbb{E}_\delta\pi^{i\theta}$ thereafter. On the other hand, if the firm stops and takes the outside option at time τ, it expects a flow payoff of $\pi^i(0,d^{-i};\mu)$ thereafter. Hence, the marginal benefit of stopping and implementing the idea is $\mathbb{E}_\delta\pi^{i\theta}-\pi^i(0,d^{-i};\mu)+c$. In contrast if the firm continues doing R&D, it obtains an expected flow payoff of $\pi^i(0,d^{-i};\mu)-c$.

Observe that firm i's payoff can be written as

$$\begin{aligned}
&\frac{1}{r}\left(\pi^i(0,d^{-i};\mu)-c+\mathbb{E}_\delta\left[e^{-r\,\tau}\left(d^i\,(\pi^{i\theta}(1,d^{-i};\mu)-\pi^i(0,d^{-i};\mu)+c)+(1-d^i)\,c\right)\right]\right)\\
=&\frac{1}{r}\left(\pi^i(0,d^{-i};\mu)-c+\mathbb{E}_\delta\left[e^{-r\,\tau}\left[\mathbb{E}_{\delta_\tau}\left(d^i\,(\pi^{i\theta}(1,d^{-i};\mu)-\pi^i(0,d^{-i};\mu)+c)+(1-d^i)\,c\right)\right]\right]\right)\\
=&\frac{1}{r}\left(\pi^i(0,d^{-i};\mu)-c+\mathbb{E}_\delta\left[e^{-r\,\tau}\left(d^i\,(\mathbb{E}_{\delta_\tau}\pi^{i\theta}(1,d^{-i};\mu)-\pi^i(0,d^{-i};\mu)+c)+(1-d^i)\,c\right)\right]\right).
\end{aligned}$$

From this it follows that for given stopping time τ, $d^{i\star}=\mathbb{I}\big(\pi^i(0,d^{-i};\mu)\le\mathbb{E}_{\delta_\tau}\pi^{i\theta}(1,d^{-i};\mu)\big)$.

Let's define the function $V(\delta)$ as the maximum between the expected present value of profits when the idea is implemented immediately and that when it is discarded immediately, both minus the present

[10] Specifically, $\mathbb{E}_\delta[\cdot]:=(1-\delta)\,\pi^{iG}+\delta\,\pi^{iB}$.

value of conducting R&D forever, conditional on the belief being δ and competitors' innovation profile being d^{-i}. It readily follows from the previous discussion that the firm's profits are

$$V(\delta) =: \frac{1}{r}\Big(\pi^i(0,d^{-i};\mu) - c + \max\left\{\delta^i\pi^{iB}(1,d^{-i};\mu) + (1-\delta^i)\pi^{iG}(1,d^{-i};\mu) - \pi^i(0,d^{-i};\mu) + c, c\right\}\Big). \tag{3}$$

Given the optimal decision $d^{i\star}$, firm i's optimal stopping problem is given by

$$\mathcal{V}(\delta) = \sup_{\tau^i \in \mathbb{T}} \left\{\mathbb{E}_\delta\left[e^{-r\,\tau^i} V(\delta_{\tau^i})\right]\right\} \tag{4}$$

subject to

$$\mathrm{d}\delta_t = \delta_t\,(1-\delta_t)\,\sigma\,\mathrm{d}B_t \quad \text{and} \quad \delta_0 = \delta.$$

Let's denote the optimal solution to (4) by $(\tau^{i\star}, d^{i\star})$. In the rest of this section, we omit the supra-index i when there is no risk of confusion since it is the only firm with an idea to be implemented.

The following is an immediate consequence of the definition of $V(\delta_\tau)$.

Lemma 1.

i) If $\pi^{iB}(1,d^{-i};\mu) - \pi^i(0,d^{-i};\mu) \geq 0$, it is optimal to implement the innovation immediately.

ii) If $\pi^{iG}(1,d^{-i};\mu) - \pi^i(0,d^{-i};\mu) < 0$, then it is optimal to discard the innovation immediately.

Proof: The proof of this and other results are relegated to the Appendix. □

According to Lemma 1, the firm i's problem admits a trivial solution in the cases considered in the Lemma. For this reason, in what follows we will restrict profits to those satisfying the following condition.

Assumption 2. *For all $d^{-i} \in \{0,1\}^{n-1}$, $\pi^{iG}(1,d^{-i};\mu) > \pi^i(0,d^{-i};\mu) > \pi^{iB}(1,d^{-i};\mu)$.*

This implies that when the idea can be easily implemented with probability one, it is profitable to do so, while when it is hard to implement with probability one, it is not worthwhile to do so.

We solve the optimal stopping problem (4) using a quasi-variational inequality (QVI) approach similar to Araman and Caldentey (2022). To this end, let us define the set of continuously differentiable functions

$$\widehat{\mathcal{C}}^2 := \left\{ f \in \mathcal{C}^1[0,1] \;:\; f''(\delta) \text{ exists } \forall \delta \in [0,1] \setminus N(f) \text{ for some finite set } N(f) \subseteq [0,1] \right\} \tag{5}$$

and the operator $\mathcal{H}$ on $\widehat{\mathcal{C}}^2$

$$(\mathcal{H}f)(\delta) := \frac{1}{2}\,\sigma^2\,\delta^2\,(1-\delta)^2\,f''(\delta) - r\,f(\delta), \qquad \text{for all } \delta \in [0,1] \setminus N(f). \tag{6}$$

Definition 1. *The function $f \in \widehat{\mathcal{C}}^2$ satisfies the quasi-variational inequalities for the agent's optimal stopping problem in (4), if for all $\delta \in [0,1] \setminus N(f)$*

$$\begin{aligned} & f(\delta) - V(\delta) \geq 0 \\ & (\mathcal{H}f)(\delta) \leq 0 \\ & \big(f(\delta) - V(\delta)\big)\,(\mathcal{H}f)(\delta) = 0. \qquad \square \end{aligned} \tag{QVI}$$

For every solution $f \in \widehat{\mathcal{C}}^2$ of the (QVI) conditions, we associate a stopping time τ_f given by

$$\tau_f = \inf\big\{t > 0 \colon f(\delta_t) = V(\delta_t)\big\}.$$

Theorem 1. (Verification) *Let $f \in \widehat{\mathcal{C}}^2$ be a solution of (QVI). Then,*

$$f(\delta) \geq \mathcal{V}(\delta) \quad \textit{for every } \delta \in [0,1].$$

In addition, if there exists a control τ_f associated with f such that $\mathbb{E}[\tau_f] < \infty$, then τ_f is optimal and $f(\delta) = \mathcal{V}(\delta)$.

According to the previous result, at optimality, the QVI conditions partition the interval $[0,1]$ into a *continuation region* where $f(\delta) > V(\delta)$ and an *intervention region* where $f(\delta) = V(\delta)$. To find a solution, we take full advantage of the fact that the payoff function $V(\delta)$ is a piecewise linear continuous function of $\delta \in [0,1]$. Moreover, $V(\delta)$ has only two linear pieces.

In the intervention region, the third QVI condition implies that $\mathcal{V}(\delta)$ solves $(\mathcal{H}\mathcal{V})(\delta) = 0$, that is,

$$\frac{(\sigma\,\delta\,(1-\delta))^2}{2}\,\mathcal{V}''(\delta) - r\,\mathcal{V}(\delta) = 0. \tag{7}$$

The two independent solutions to this ODE are given by $F(\delta)$ and $F(1-\delta)$ with

$$F(\delta) \equiv \frac{(1-\delta)^\gamma}{\delta^{\gamma-1}} \quad \text{where } \gamma \equiv \frac{1 + \sqrt{1 + 8\,r/\sigma^2}}{2} \tag{8}$$

and the general solution to (7) is

$$\mathcal{V}(\delta) = \beta_0\, F(\delta) + \beta_1\, F(1-\delta), \tag{9}$$

where β_0 and β_1 are the constants of integration, whose values are determined by imposing value-matching $\mathcal{V}(\delta) = 1$ and smooth-pasting condition $\mathcal{V}_\delta(\delta) = 0$ at $\delta = \bar{\delta}$.

It will be useful to define the function

$$\widehat{\mathcal{V}}(\delta; \bar{\delta}) \equiv \begin{cases} \frac{(\gamma-\bar{\delta})}{(2\,\gamma-1)}\,\frac{F(\delta)}{F(\bar{\delta})} + \frac{(\gamma+\bar{\delta}-1)}{(2\,\gamma-1)}\,\frac{F(1-\delta)}{F(1-\bar{\delta})} & \text{if} \quad 0 < \delta \leq \bar{\delta}^\star \\ 1 & \text{if} \quad \bar{\delta}^\star < \delta \leq 1 \end{cases} \tag{10}$$

This corresponds to the solution to equation (7) imposing value-matching and smooth-pasting at $\bar{\delta}$. Thus, $\widehat{\mathcal{V}}(\delta; \bar{\delta})$ is decreasing and strictly convex in $\delta \in (0,1)$ for a fixed $\bar{\delta}$. Also, $\widehat{\mathcal{V}}(\delta; \bar{\delta})$ increases with $\bar{\delta}$ for a given δ. This function is fundamental to understanding the problem because it captures the benefit of conducting R$\hat{\text{D}}$. Its convexity tells us that the benefit of conducting R&D increases in the initial belief that the idea is good.

The following result follows from the previous discussion.

Proposition 1. *Suppose Assumption 2 holds. Then, firm i's expected profit is given by*

$$\mathcal{V}(\delta) = \frac{1}{r} \begin{cases} \delta \pi^{iB}(1, d^{-i}; \mu) + (1-\delta)\pi^{iG}(1, d^{-i}; \mu) & \text{if} \quad 0 \leq \delta \leq \underline{\delta}^\star, \\ \widehat{\mathcal{V}}(\delta; \bar{\delta})\pi^i(0, d^{-i}; \mu) & \text{if} \quad \underline{\delta}^\star < \delta < \bar{\delta}^\star, \\ \pi^i(0, d^{-i}; \mu) & \text{if} \quad \bar{\delta}^\star \leq \delta \leq 1, \end{cases} \tag{11}$$

and the thresholds $\underline{\delta}^\star$ and $\bar{\delta}^\star$ are determined imposing value-matching ($\mathcal{V}(\delta) = V(\delta)$) and smooth-pasting ($\mathcal{V}_\delta(\delta) = V_\delta(\delta)$) conditions at $\delta = \underline{\delta}^\star$ and $\delta = \bar{\delta}^\star$, and satisfy

$$\underline{\delta}^\star < (\pi^{iG}(1, d^{-i}; \mu) - \pi^i(0, d^{-i}; \mu))/(\pi^{iG}(1, d^{-i}; \mu) - \pi^{iB}(1, d^{-i}; \mu)) < \bar{\delta}^\star.$$

The profit-maximizing strategy $(\tau^\star, d^\star)$ is given by

$$\tau^\star = \inf\left\{t > 0\colon \delta_t \notin (\underline{\delta}^\star, \bar{\delta}^\star)\right\} \qquad \textit{and} \qquad d^\star = \mathbb{I}(\delta_{\tau^\star} \leq \underline{\delta}^\star).$$

The problem of firm i is that the only relevant information for its decision each time is the current belief. The trajectory of the belief is irrelevant due to the updating process's martingale nature and the objective function's quasi-linearity. This allows us to characterize the firm i's problem as an optimal hitting time with a high and a low threshold.

When the posterior hits the high threshold, it is profit-maximizing to discard the idea, while when it hits the low threshold, it is profit-maximizing to implement the idea.

Because the firm can stop R&D at any time, the expected incremental profits must be large enough so that the firm is willing to keep conducting R&D instead of discarding the idea. This happens when the firm believes the idea is hard with high probability (i.e., $\delta \geq \bar{\delta}^\star$) since its expected present value is lower than the present value of keep producing with the old technology. When the firm stops –the posterior hits the high threshold– and discards the idea, the value-matching and smooth-pasting conditions must be satisfied. Otherwise, there would be room for improvement (see, Figure 1).

Similarly, since firm i can implement the innovation whenever it wants, to be optimal for it to keep the idea as an option and to stay learning, it must be optimal for the firm to postpone it. This happens when the firm believes the idea is easy with high probability (i.e., $\delta \leq \underline{\delta}^\star$) since its expected present value from producing with the new technology exceeds that from producing with the current technology. When the firm stops –the posterior hits the low threshold– and implements the idea, the value-matching and smooth-pasting conditions must be satisfied. Again, if this is not met, there is room for improvement (see, Figure 1).

Because waiting to get the return to the idea when implemented or saving the R&D cost when discarded is costly, when the prior is neither high nor low (i.e. $\underline{\delta}^\star < \delta < \bar{\delta}^\star$), the expected discounted profits upon reaching time $t \leq \tau^\star$ must be larger than the maximum between $\mathbb{E}_\delta \pi^{i\theta}(1, d^{-i}; \mu)$ and $(\pi^i(0, d^{-i}; \mu)$ to compensate for the extra time it will take to get this due to R&D (see Figure 1).

We next provide a probabilistic characterization of the firm i's optimal strategy $(\tau^\star, d^\star)$ when R&D is conducted. The result is based on the dynamics of the belief process δ_t, as detailed in Equation (1), the hitting time representation of $\tau^\star$ in Proposition 1, and Dynkin's formula (see Øksendal, 2013).

Proposition 2. *Suppose $\delta \in (\underline{\delta}^\star, \bar{\delta}^\star)$, then the optimal stopping $\tau^\star$ has the moment generating function*

$$\mathbb{E}_\delta[e^{s\,\tau^\star}] = \frac{\left(F(1-\bar{\delta}^\star) - F(1-\underline{\delta}^\star)\right) F(\delta) + \left(F(\underline{\delta}^\star) - F(\bar{\delta}^\star)\right) F(1-\delta)}{F(\underline{\delta}^\star)\, F(1-\bar{\delta}^\star) - F(\bar{\delta}^\star)\, F(1-\underline{\delta}^\star)}.$$

The expected amount of time firm i spends conducting R&D is equal to

$$\mathbb{E}_\delta[\tau^\star] = \left(\frac{\bar{\delta}^\star - \delta}{\bar{\delta}^\star - \underline{\delta}^\star}\right) g(\underline{\delta}^\star) + \left(\frac{\delta - \underline{\delta}^\star}{\bar{\delta}^\star - \underline{\delta}^\star}\right) g(\bar{\delta}^\star) - g(\delta), \quad \textit{where} \quad g(\delta) = \frac{2\,(1-2\delta)}{\sigma^2} \ln\left(\frac{1-\delta}{\delta}\right).$$

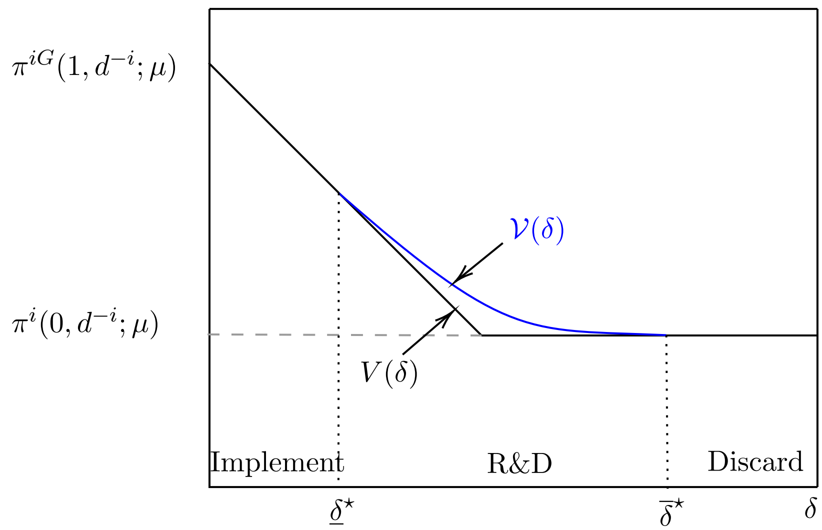

Figure 1: firm i's expected discounted payoff $\mathcal{V}(\delta)$ as a function of the belief δ. The range of beliefs is partition into three regions: (i) for $\delta \in [0, \underline{\delta}^{\star}]$ the firm implements the idea, (ii) for $\delta \in (\underline{\delta}^{\star}, \bar{\delta}^{\star})$ the firm conducts R&D and (iii) for $\delta \in [\bar{\delta}^{\star}, 1]$ the firm discard the idea.

The probability that firm i implements its innovation and the probability that it does not are given by

$$\mathbb{P}_{\delta}(d^{\star} = 1) = \frac{\bar{\delta}^{\star} - \delta}{\bar{\delta}^{\star} - \underline{\delta}^{\star}} \qquad \textit{and} \qquad \mathbb{P}_{\delta}(d^{\star} = 0) = \frac{\delta - \underline{\delta}^{\star}}{\bar{\delta}^{\star} - \underline{\delta}^{\star}}, \quad \textit{respectively.}$$

In the remainder of this section, we will study the effect of competition on the three different but related measures of R&D intensity. One is the difference between the high and the low threshold, $\bar{\delta}^{*} - \underline{\delta}^{*}$. Another is the expected amount of time the firm spends conducting R&D. The other is the probability the idea is implemented.

The first measure of competitiveness is the regular profits, i.e., profits the firm makes while conducting R&D or when the idea is discarded. The smaller the regular profits, the more competitive the market pre-innovation. This speaks directly to Arrow's replacement effect.

Proposition 3 (Arrow's Replacement Effect)**.**

i) $\bar{\delta}^{\star}$, $\underline{\delta}^{\star}$, *and* $\bar{\delta}^{\star} - \underline{\delta}^{\star}$ *fall with* $\pi^{i}(0, d^{-i}; \mu)$.

ii) The probability that the idea is implemented falls with $\pi^{i}(0, d^{-i}; \mu)$.

iii) The expected R&D intensity, $\mathbb{E}_{\delta}[\tau^{\star}]$*, falls with* $\pi^{i}(0, d^{-i}; \mu)$.

On the one hand, the larger the regular profits, i.e., the less competitive the market pre-innovation, the less costly it is to discard the idea since the incremental profits are smaller. On the other hand, the cost of conducting R&D is smaller since while doing it, the firm gets $\pi^i(0, d^{-i}; \mu) - c$. Thus, implementing the idea requires a higher standard regarding the experiment results, i.e., a lower threshold below which the idea is implemented.

Thus, an increase in regular profits not only gives rise to Arrow's replacement effect but also to less experimentation and, therefore, less information regarding the quality of the idea.

Next, we study how R&D and the probability that the innovation is implemented are affected by the competition intensity, as measured by μ. This exercise differs from the preceding one since competition intensity lowers regular profits and those from innovation, and thereby, more intense competition may either increase or decrease the incremental profits from innovation. Thus, more intense competition results not only in a replacement effect but also in a reducing-profitability effect.

Assumption 3.

$$\frac{\pi^{iG}_{\mu}}{\pi^{iG}} \geq \frac{\pi^{i}_{\mu}}{\pi^{i}} \geq \frac{\pi^{iD}_{\mu}}{\pi^{iD}}.$$

Proposition 4. *Suppose that Assumption 3 holds.*

i) If

$$\underline{\delta}^\star \left(\pi^{iD}_{\mu} - \frac{\pi^{iD}}{\pi^{i}}\pi^{i}_{\mu}\right) + (1 - \underline{\delta}^\star)\left(\pi^{iG}_{\mu} - \frac{\pi^{iG}}{\pi^{i}}\pi^{i}_{\mu}\right) \leq 0, \tag{12}$$

then $(\bar{\delta}^\star, \underline{\delta}^\star)$*.* $\bar{\delta}^\star - \underline{\delta}^\star$*, and* $\mathbb{P}_{\bar{\delta}}(d^\star = 1)$ *rise with* μ*.*

ii) If the inequality in equation (12) does not hold, then $\bar{\delta}^\star$ *rises with* μ*,* $\underline{\delta}^\star$ *rises with* μ *whenever* $\underline{\delta}^\star \geq 1/2$ *or* $\underline{\delta}^\star < 1/2$ *and*

$$\underline{\delta}^\star G_D(\gamma, \bar{\delta}^\star, \underline{\delta}^\star)\left(\pi^{iD}_{\mu} - \frac{\pi^{iD}}{\pi^{i}}\pi^{i}_{\mu}\right) + (1 - \underline{\delta}^\star) G_E(\gamma, \bar{\delta}^\star, \underline{\delta}^\star)\left(\pi^{iG}_{\mu} - \frac{\pi^{iG}}{\pi^{i}}\pi^{i}_{\mu}\right) \geq 0, \tag{13}$$

and falls otherwise. $\bar{\delta}^\star - \underline{\delta}^\star$ *rises with* μ *and* $\mathbb{P}_{\bar{\delta}}(d^\star = 1)$ *may either rises or falls with* μ*.*

iii) If the inequalities in equations (12) and (13) do not hold, then $(\bar{\delta}^\star, \underline{\delta}^\star)$*.* $\bar{\delta}^\star - \underline{\delta}^\star$*, and* $\mathbb{P}_{\bar{\delta}}(d^\star = 1)$ *rise with* μ*.*

On the one hand, because increased competition intensity lowers regular profits, the replacement effect implies the firm implements the idea more frequently, and the high and low threshold as well as its difference rise.

On the other hand, increased competition intensity decreases the expected profits from the innovation. The decreases in π^{iD}, ceteris-paribus, implies that the discarding the project is more attractive and conducting more R&D to avoid implementing a bad idea is worthwhile.

When the expected incremental value of innovation in percentage terms is negative, the high and low threshold and its difference fall as well as the probability that the idea is implemented.

In particular, how much regular profits fall in percentage terms relative to the drop, in percentage terms, of the expected profits from innovation evaluated at the optimal thresholds. When regular profits falls less, the profit-reducing effect dominates the replacement effect and R&D and implementation suffers.

In contrast, when the opposite happens, the upper threshold rises and thereby the firm tolerates more bad news before discarding the project. However, the firm could more or less inclined to implement the innovation as good information arrive. This depends on how competition intensity affect profits when the idea is badrelative to when it is good. The firm will be more inclined to implement the idea after positive information when the profits of the good idea relative to the regular profits suffer relatively much less than the profits of the badidea relative to the regular profits. This happens because the prize of innovation, i.e., the slope of the expected profits, rises. In this case, the probability that the ideas is implemented rises with competition intensity.

The impact of a stronger competition on the intensity of R&D, as measured by $\bar{\delta}^\star - \underline{\delta}^\star$, is negative only if $\bar{\delta}^\star - 2\underline{\delta}^\star > 0$

4.2 Multiple Ideas: Sequential R&D

We extend our previous analysis by allowing multiple independent R&D ideas to be researched sequentially. Our concern relates to whether having new ideas increases the probability of either accepting or rejecting the current idea and the time spent on R&D in the initial idea being researched.

The firm can engage in R&D in a new idea once the current research idea is rejected. Once an idea is accepted and implemented, it is not possible to engage in R&D. It is as if the cost of running two parallel R&D processes is infinite. This could be due to limited financial resources or a limited number of scientists.

To keep the analysis as simple as possible, let's assume there are two ideas, denoted by g and b. Thus, idea g is better than idea b, and thereby, idea g is undertaken first. This implies that the firm chooses between two risky armed bandits instead of a risky and safe armed bandit. The riskiness of the new arm bandit is endogenous since if it is rejected, the payoff is the same as when the first idea is rejected.

The only difference between idea g and idea b is the second-order differential equation governing the R&D process. Namely, the SDE governing the R&D in idea g is given by that in equation (1), whereas the SDE governing the R&D in idea b is as follows

$$d\eta_t = \eta_t\,(1-\eta_t)\,\mathrm{d}A_t \quad \text{and} \quad A_t = \sigma\,\rho B_t + \sigma_b\,\sqrt{1-\rho^2}C_t, and\, \eta_0 = \eta\,\text{otherwise.} \tag{14}$$

where B_t and C_t are two independent Brownian motions. Thus, A_t and B_t are correlated with correlation coefficient $\rho \in [-1, 1]$, where $\eta_t = \mathbb{P}(\eta = 1 \mid \mathcal{G}_t)$ denote the agent's belief after conducting due diligence for t time units, and $\mathcal{G}_t$ denote the usual filtration generated by A_t. Let $\mathbb{T}$ be the set of stopping times regarding $\mathbb{G}$. We will naturally require $\tau_b \in \mathbb{T}$ and $d_b \in \mathcal{G}_\tau$.

Let's define the function

$$V(\eta) \equiv \pi^i(0, d^{-i}; \mu) - c + \max\left\{\eta\pi^{iD}(1, d^{-i}; \mu) + (1-\eta)\pi^{iG}(1, d^{-i}; \mu) - \pi^i(0, d^{-i}; \mu) + c, c\right\}.$$

In contrast, the firm i's optimal stopping problem is given by

$$\mathcal{V}(\eta; \rho) =: \sup_{\tau_b^i \in \mathbb{T}} \left\{\mathbb{E}_\eta\left[V(\eta_{\tau_b})\right]\right\}$$

subject to

$$\mathrm{d}\eta_t = \eta_t\,(1-\eta_t)\,(\sigma\,\rho B_t + \sigma_b\,\sqrt{1-\rho^2}C_t) \quad \text{and} \quad \eta_0 = \eta.$$

According to the verification theorem, at optimality, the QVI conditions partition the interval $[0, 1]$ into a *continuation region* where $f(\eta) > V(\eta)$ and an *intervention region* where $f(\eta) = V(\eta)$. To find a solution, we take full advantage of the payoff function $V(\delta)$ being a piecewise linear continuous function of $\eta \in [0, 1]$. Moreover, $V(\eta)$ has only two linear pieces.

In the intervention region, the third QVI condition implies that $\mathcal{V}(\eta; \rho)$ solves $(\mathcal{H}\mathcal{V})(\eta; \rho) = 0$, that is,

$$\frac{(\sigma(\rho)\,\eta\,(1-\eta))^2}{2}\,\mathcal{V}''(\eta; \rho) - r\,\mathcal{V}(\eta; \rho) = 0, \tag{15}$$

where $\sigma(\rho) \equiv \sigma^2\rho^2 + \sigma_b^2(1-\rho^2)$.

Thus, this problem has the same structure as the one we already solved in the last section. The solution is also a cutoff strategy $(\underline{\eta}_b^\star, \bar{\eta}_b^\star)$ with parameter $\sigma(\rho)$ instead of σ. The equilibrium payoff is given by

$$\mathcal{V}(\eta;\rho) = \frac{1}{r}\begin{cases} \delta\pi^{iD}(1, d^{-i};\mu) + (1-\delta)\pi^{iG}(1, d^{-i};\mu) & \text{if} \qquad 0 \le \eta \le \underline{\eta}^\star, \\ \widehat{\mathcal{V}}(\eta;\bar{\eta},\rho)\pi^i(0, d^{-i};\mu) & \text{if} \qquad \underline{\eta}^\star < \eta < \bar{\eta}^\star, \\ \pi^i(0, d^{-i};\mu) & \text{if} \qquad \bar{\eta}^\star \le \eta \le 1, \end{cases} \tag{16}$$

where

$$\widehat{\mathcal{V}}(\eta;\bar{\eta},\rho) \equiv \begin{cases} \frac{(\gamma(\rho)-\bar{\eta})}{(2\,\gamma(\rho)-1)}\,\frac{F(\eta)}{F(\bar{\eta})} + \frac{(\gamma(\rho)+\bar{\eta}-1)}{(2\,\gamma(\rho)-1)}\,\frac{F(1-\eta)}{F(1-\delta)} & \text{if} \qquad 0 < \eta \le \bar{\eta}^\star \\ 1 & \text{if} \qquad \bar{\eta}^\star < \eta \le 1 \end{cases} \tag{17}$$

The firm's payoff in period 0 is

$$\int_{\tau_g^i}^{\infty} e^{-r\,t}\left(d_g^i \mathbb{E}_\delta \pi^{i\theta}(1, d^{-i};\mu) + (1-d_g^i)\,\mathcal{V}(\eta;\rho)\right)dt + \int_0^{\tau_g^i} e^{-r\,t}\big(\pi^i(0, d^{-i};\mu) - c\big)dt. \tag{18}$$

It readily follows from this firm i's profits can be written as follows

$$\begin{aligned} V(\delta;\eta,\rho) = \frac{1}{r}\Big(\pi^i(0, d^{-i};\mu) - c + \max\Big\{&\delta^i\pi^{iD}(1, d^{-i};\mu) + (1-\delta^i)\pi^{iG}(1, d^{-i};\mu) - \pi^i(0, d^{-i};\mu) + c, \\ &\mathcal{V}(\eta;\rho) - \pi^i(0, d^{-i};\mu) + c\Big\}\Big). \end{aligned}$$

Given the optimal decision $d_g^{i\star}$, firm i's optimal stopping problem is given by

$$\mathcal{V}(\delta;\eta,\rho) = \sup_{\tau_g^i \in \mathbb{T}} \left\{\mathbb{E}_\delta\left[e^{-r\,\tau_g^i} V(\delta_{\tau_g^i};\eta)\right]\right\} \tag{19}$$

subject to

$$\mathrm{d}\delta_t = \delta_t\,(1-\delta_t)\,\sigma\,\mathrm{d}B_t \quad \text{and} \quad \delta_0 = \delta.$$

It will become useful to define the function

$$\widehat{\mathcal{V}}(\delta;\bar{\delta}) \equiv \frac{(\gamma-\bar{\delta})}{(2\,\gamma-1)}\,\frac{F(\delta)}{F(\bar{\delta})} + \frac{(\gamma+\bar{\delta}-1)}{(2\,\gamma-1)}\,\frac{F(1-\delta)}{F(1-\bar{\delta})} \quad \text{if} \qquad 0 < \delta < \bar{\delta}^\star. \tag{20}$$

This corresponds to the solution to equation (7) imposing value-matching and smooth-pasting at $\bar{\delta}$. Thus, $\widehat{\mathcal{V}}(\delta;\bar{\delta},\eta,\rho)$ is decreasing and strictly convex in $\delta \in (0,1)$ for a fixed $\bar{\delta}$. Also, $\widehat{\mathcal{V}}(\delta;\bar{\delta},\eta,\rho)$ increases with $\bar{\delta}$ for a given δ.

The following result follows from the previous discussion and Proposition 1

Proposition 5. *Suppose Assumption 2 holds. Then, firm i's expected profit is*

$$\mathcal{V}(\delta;\eta,\rho)=\frac{1}{r}\begin{cases}\delta\pi^{iD}(1,d^{-i};\mu)+(1-\delta)\pi^{iG}(1,d^{-i};\mu) & \text{if}\quad 0\le\delta\le\underline{\delta}^\star,\\ \widehat{\mathcal{V}}(\delta;\bar{\delta})\mathcal{V}(\eta;\rho) & \text{if}\quad \underline{\delta}^\star<\delta<\bar{\delta}^\star,\\ \mathcal{V}(\eta;\rho) & \text{if}\quad \bar{\delta}^\star\le\delta\le 1,\end{cases}\tag{21}$$

and the thresholds $\underline{\delta}^\star$ and $\bar{\delta}^\star$ are determined imposing value-matching and smooth-pasting conditions at $\delta=\underline{\delta}^\star$ and $\delta=\bar{\delta}^\star$, and satisfy $\underline{\delta}^\star<(\pi^{iG}(1,d^{-i};\mu)-\mathcal{V}(\eta;\rho))/(\pi^{iG}(1,d^{-i};\mu)-\pi^{iD}(1,d^{-i};\mu))<\bar{\delta}^\star$.

The profit-maximizing strategy $(\tau_g^\star, d_g^\star)$ is given by

$$\tau_g^\star=\inf\left\{t>0\colon \delta_t\notin(\underline{\delta}^\star,\bar{\delta}^\star)\right\}\qquad\text{and}\qquad d_g^\star=\mathbb{I}(\delta_{\tau_g^\star}\le\underline{\delta}^\star).$$

Proposition 6. *Suppose that $\sigma>\sigma_b$. Then, as the correlation between the Browning motions rises, the probability of rejecting the first idea increases whenever $\bar{\delta}^\star\le 1/2$.*

5 Two Innovators with Sequential R&D

In this section, we discuss the case in which two firms, denoted by 1 and 2, each having an idea that it wants to implement but before doing so each may engage in R&D.

We assume that firm 1 engages in R&D first and firm 2 does so after observing firm 1's decisions and the outcome of its innovation when implemented. Firm i's profits depend on whether firm j's idea is implemented or not and the quality of it as follows.

Assumption 4.

i) For $i\in\{1,2\}$ and $\theta\in\{D,E\}$, $\pi^{i\theta}(1,1;\mu)-\pi^{i}(0,1;\mu)\lesseqgtr\pi^{i\theta}(1,0;\mu)-\pi^{i}(0,0;\mu)$.

ii) For $i,j\in\{1,2\}$, $i\neq j$ and $d^j\in\{0,1\}$, $\pi^{iG}(1,1;\mu)-\pi^{iG}(1,0;\mu)>\pi^{iD}(1,1;\mu)-\pi^{iD}(1,0;\mu)$.

iii) For $i\in\{1,2\}$ and $\theta\in\{D,E\}$, $\pi^{i\theta}(d^i,1;\mu)-\pi^{i\theta}(d^i,0;\mu)<0$ and $\pi^{i}(0,1;\mu)-\pi^{i}(0,0;\mu)<0$.

Part i) says that profits from the innovation can be complements or substitutes. Part ii) assumes that firm i's innovation increases its profit more when the idea is easier than when it is badfor any investment decision for firm j. Part iii) establishes that firm i's profits fall when firm j implements its idea regardless of its quality.

Firm 2's optimization problem is identical to the one solved in Section 4. Let firm 2's prior be $\eta \in (0,1)$. It readily follows from Proposition 1 that firm 2's optimal strategy is a threshold strategy with two thresholds, denoted by $\bar{\eta}(d^1)$ and $\underline{\eta}(d^1)$, where $d^1 = 1$ when firm 1 implement its idea and $d^1 = 0$ when it does not.

It readily follows from Proposition 2 that the probability that firm 2 implements its idea is given by

$$\mathbb{P}_\delta(d^{2\star} = 1|d^1) = \frac{\bar{\eta}^\star(d^1) - \eta}{\bar{\eta}^\star(d^1) - \underline{\eta}^\star(d^1)}.$$

The probability that the idea is implemented when $d^1 = 1$ is larger than that when $d^1 = 0$ whenever $(\eta - \underline{\eta}^\star(0))(\bar{\eta}^\star(1) - \bar{\eta}^\star(0)) > (\eta - \bar{\eta}^\star(0))(\underline{\eta}^\star(1) - \underline{\eta}^\star(0))$.

Proposition 7. *If*

$$\frac{\pi^{2D}(1,1;\mu) - \pi^{2E}(1,1;\mu))}{\pi^2(0,1;\mu)} > \frac{\pi^{2D}(1,0;\mu) - \pi^{2E}(1,0;\mu))}{\pi^2(0,0;\mu)} \text{ and } \frac{\pi^{2E}(1,1;\mu))}{\pi^2(0,1;\mu)} > \frac{\pi^{2E}(1,0;\mu))}{\pi^2(0,0;\mu)}, \tag{22}$$

, then $\mathbb{P}_\delta(d^{2\star} = 1|1) > \mathbb{P}_\delta(d^{2\star} = 1|0)$.

Thus, firm 2's probability to implement its idea is larger when firm 1 implements its than when it does not whenever i) the ratio of the difference between firm 2's profits when the idea is badand the profits when it is good to the regular profits is larger when firm 1 innovates and ii) the ratio of the good idea profit to the regular profits is larger when firm 1 innovates. In other words, when firm 1's innovation hits less harder firm 2's profits gain from its idea than it hits regular profits. Hence, in this case, firm 1's competition induces firm 2 to do more R&D and to implement its idea more frequently.

Given this, firm 1's profits are

$$\begin{aligned} V(\delta;\underline{\eta}^\star,\bar{\eta}^\star) =:& \frac{1}{r}\Big(\mathbb{E}_{d^{2\star}}[\pi^1(0,d^{2\star};\mu)] - c + \\ & \max\left\{\delta\mathbb{E}_{d^{2\star}}[\pi^{1D}(1,d^{2\star};\mu)] + (1-\delta)\mathbb{E}_{d^{2\star}}[\pi^{1E}(1,d^{2\star};\mu)] - \mathbb{E}_{d^{2\star}}[\pi^1(0,d^{2\star};\mu)] + c, c\right\}\Big), \end{aligned}$$

where $\mathbb{E}_{d^{2\star}}[\pi^{1\theta}(1,d^2;\mu)] =: \mathbb{P}_\delta(d^{2\star} = 1|1)\pi^{1\theta}(1,1;\mu) + (1 - \mathbb{P}_\delta(d^{2\star} = 1|1))\pi^{1\theta}(1,0;\mu)$ for $\theta \in \{E,D\}$ and $\mathbb{E}_{d^{2\star}}[\pi^1(1,d^2;\mu)] =: \mathbb{P}_\delta(d^{2\star} = 1|0)\pi^1(0,1;\mu) + (1 - \mathbb{P}_\delta(d^{2\star} = 1|0))\pi^1(0,0;\mu)$.

It follows from Assumption 4 that in each possible state, firm 1's profits are smaller when firm 2 implements its idea. Thus, a priori, the fact that firm 1 faces competition in the future may either increase or decrease its incentives to carry R&D and to implement its idea.

Given the optimal decision $d^{1\star}$, firm i's optimal stopping problem is given by

$$\mathcal{V}(\delta, \underline{\eta}^\star, \bar{\eta}^\star) = \sup_{\tau^i \in \mathbb{T}} \left\{ \mathbb{E}_\delta \left[e^{-r\,\tau^i} V(\delta_{\tau^i}; \underline{\eta}^\star, \bar{\eta}^\star) \right] \right\} \tag{23}$$

subject to

$$\mathrm{d}\delta_t = \delta_t\,(1-\delta_t)\,\sigma\,\mathrm{d}B_t \quad \text{and} \quad \delta_0 = \delta.$$

This problem is also identical to the one solved in Section 4 but for the fact that now the expected profits with respect δ and firm 2's optimal strategy, which depends on what firm 1 does, replace the expected profits with respect only to δ.

It readily follows from Proposition 1 that firm 1's optimal strategy is a threshold strategy with two thresholds, denoted as before, by $\bar{\delta}^{\star,c}$ and $\underline{\delta}^{\star,c}$, where the c stands for competition.

When the probability that firm 2 implements its idea rises when firm 1 implements its idea, firm 1's expected profits from implementing its idea fall and regular profits also falls. Thus, the firm 1's incentives to carry R&D and to implement its idea may either rise or fall.

Using the result in Proposition 4, we can conclude the following:

Proposition 8. *Suppose that the condition in Proposition 22 holds. Then,*

6 Concluding Remarks

This paper provides an answer to a classical question, which is whether competition increases R&D or not. The answer is, as in most cases in this topic, ambiguous. However, the paper highlights that what happens depends on how strong is Arrow's replacement effect relative to the profit-reducing effect. That is the impact of competition intensity on the innovation's expected profits when these are evaluated at the optimal stopping time.

References

Philippe Aghion and Patrick Bolton. A theory of trickle-down growth and development. *Review of Economic Studies*, 64:151–172, 1997. doi: https://doi.org/10.2307/2971707. 5

Philippe Aghion and Peter Howitt. A model of growth through creative destruction. *Econometrica*, 60(2):323–51, 1992. URL https://EconPapers.repec.org/RePEc:ecm:emetrp:v:60:y:1992:i:2:p:323-51. 2

Philippe Aghion, Mathias Dewatripont, and Patrick Rey. Competition, financial discipline and growth. *The Review of Economic Studies*, 66(4):825–852, 1999. 2

Philippe Aghion, Christopher Harris, Peter Howitt, and John Vickers. Competition, imitation and growth with step-by-step innovation. *Review of Economic Studies*, 68(3):467–492, 2001. URL https://EconPapers.repec.org/RePEc:oup:restud:v:68:y:2001:i:3:p:467-492. 5

Philippe Aghion, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt. Competition and Innovation: an Inverted-U Relationship. *The Quarterly Journal of Economics*, 120(2):701–728, 2005. URL https://ideas.repec.org/a/oup/qjecon/v120y2005i2p701-728..html. 5, 6, 7

Phillipe Aghion, Mathias Dewatripont, and Patrick Rey. Corporate governance, competition policy and industrial policy. *European Economic Review*, 41(3-5):797–805, 1997. 2

Rabah Amir. Cournot oligopoly and the theory of supermodular games. *Games and Economic Behavior*, 15(2):132–148, 1996. URL https://EconPapers.repec.org/RePEc:eee:gamebe:v:15:y:1996:i:2:p:132-148. 10

V.F. Araman and R.A. Caldentey. Diffusion approximations for a class of sequential experimentation problems. *Management Science*, 68(8):5958–5979, 2022. 2, 4, 11, 13

Kenneth Arrow. Economic welfare and the allocation of resources for invention. In *The Rate and Direction of Inventive Activity: Economic and Social Factors*, pages 609–626. National Bureau of Economic Research, Inc, 1962. URL https://EconPapers.repec.org/RePEc:nbr:nberch:2144. 2, 5, 8

Susan Athey and Armin Schmutzler. Investment and Market Dominance. *RAND Journal of Economics*, 32(1):1–26, Spring 2001. URL https://ideas.repec.org/a/rje/randje/v32y2001i1p1-26.html. 3, 5, 9

Martin Neil Baily, Hans Gersbach, FM Scherer, and Frank R Lichtenberg. Efficiency in manufacturing and the need for global competition. *Brookings Papers on Economic Activity. Microeconomics*, 1995: 307–358, 1995. 2, 5, 7

Pilar Beneito, Paz Coscollá-Girona, María Engracia Rochina-Barrachina, and Amparo Sanchis. Competitive pressure and innovation at the firm level. *The Journal of Industrial Economics*, 63(3): 422–457, 2015. doi: 10.1111/joie.12079. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/joie.12079. 7

Nicholas Bloom, Mark Schankerman, and John Van Reenen. Identifying technology spillovers and product market rivalry. *Econometrica*, 81(4):1347–1393, 2013. ISSN 1468-0262. doi: 10.3982/ECTA9466. URL http://dx.doi.org/10.3982/ECTA9466. 10

Richard Blundell, Rachel Griffith, and John van Reenen. Dynamic count data models of technological innovation. *Economic Journal*, 105(429):333–44, 1995. URL https://EconPapers.repec.org/RePEc:ecj:econjl:v:105:y:1995:i:429:p:333-44. 7

Jan Boone. Competitive Pressure: The Effects on Investments in Product and Process Innovation. *RAND Journal of Economics*, 31(3):549–569, Autumn 2000. URL https://ideas.repec.org/a/rje/randje/v31y2000iautumnp549-569.html. 3, 6, 9

Jan Boone. Intensity of competition and the incentive to innovate. *International Journal of Industrial Organization*, 19(5):705–726, April 2001. URL https://ideas.repec.org/a/eee/indorg/v19y2001i5p705-726.html. 6, 9

Fernando Branco, Monic Sun, and J Miguel Villas-Boas. Optimal search for product information. *Management Science*, 58(11):2037–2056, 2012. 5

Ricardo J. Caballero and Adam B. Jaffe. How High are the Giants' Shoulders: An Empirical Assessment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth. In *NBER Macroeconomics Annual 1993, Volume 8*, NBER Chapters, pages 15–86. National Bureau of Economic Research, Inc, June 1993. URL https://ideas.repec.org/h/nbr/nberch/10998.html. 2

H. Chernoff. Sequential design of experiments. *Ann. Math. Statist.*, 30(3):755–770, 09 1959. doi: 10.1214/aoms/1177706205. 4

H. Chernoff. *Sequential Analysis and Optimal Design*. SIAM, Philadelphia, PA, 1972. 4

Claude d'Aspremont and Alexis Jacquemin. Cooperative and noncooperative r&d in duopoly with spillovers. *American Economic Review*, 78(5):1133–37, 1988. URL https://EconPapers.repec.org/RePEc:aea:aecrev:v:78:y:1988:i:5:p:1133-37. 10

L. Fan and P. W. Glynn. Diffusion approximations for thompson sampling. Working paper, Management Science and Engineering, Stanford University, 2021. URL https://arxiv.org/pdf/2105.09232.pdf. 5

Drew Fudenberg, Philipp Strack, and Tomasz Strzalecki. Speed, accuracy, and the optimal timing of choices. *American Economic Review*, 108(12):3651–3684, 2018. 5

Richard Gilbert, Christian Riis, and Erlend S. Riis. Stepwise innovation by an oligopoly. *International Journal of Industrial Organization*, 61:413 – 438, 2018. ISSN 0167-7187. doi: https://doi.org/10.1016/j.ijindorg.2018.10.001. URL http://www.sciencedirect.com/science/article/pii/S0167718718300997. 6

Ronald L. Goettler and Brett R. Gordon. Does amd spur intel to innovate more? *Journal of Political Economy*, 119(6):1141–1200, 2011. doi: 10.1086/664615. URL https://doi.org/10.1086/664615. 7

Gene M Grossman and Elhanan Helpman. Endogenous Product Cycles. *Economic Journal*, 101(408):1214–1229, September 1991. URL https://ideas.repec.org/a/ecj/econjl/v101y1991i408p1214-29.html. 2

Bronwyn H Hall and Josh Lerner. The financing of r&d and innovation. In *Handbook of the Economics of Innovation*, volume 1, pages 609–639. Elsevier, 2010. 8

Oliver D. Hart. The Market Mechanism as an Incentive Scheme. *Bell Journal of Economics*, 14(2):366–382, Autumn 1983. URL https://ideas.repec.org/a/rje/bellje/v14y1983iautumnp366-382.html. 2

Aamir Rafique Hashmi. Competition and Innovation: The Inverted-U Relationship Revisited. *The Review of Economics and Statistics*, 95(5):1653–1668, 12 2013. ISSN 0034-6535. doi: 10.1162/REST_a_00364. URL https://doi.org/10.1162/REST_a_00364. 7

Aamir Rafique Hashmi and Johannes Van Biesebroeck. The Relationship between Market Structure and Innovation in Industry Equilibrium: A Case Study of the Global Automobile Industry. *The Review of Economics and Statistics*, 98(1):192–208, 03 2016. ISSN 0034-6535. doi: 10.1162/REST_a_00494. URL https://doi.org/10.1162/REST_a_00494. 7

Mitsuru Igami. Estimating the innovators dilemma: Structural analysis of creative destruction in the hard disk drive industry, 19811998. *Journal of Political Economy*, 125(3):798–847, 2017. doi: 10.1086/691524. URL https://doi.org/10.1086/691524. 8

Mitsuru Igami and Kosuke Uetake. Mergers, Innovation, and Entry-Exit Dynamics: Consolidation of the Hard Disk Drive Industry, 19962016. *The Review of Economic Studies*, 87(6):2672–2702, 09 2019. ISSN 0034-6527. doi: 10.1093/restud/rdz044. URL https://doi.org/10.1093/restud/rdz044. 8

William R. Kerr and Ramana Nanda. Financing innovation. *Annual Review of Financial Economics*, 7(1):445–462, 2015. doi: 10.1146/annurev-financial-111914-041825. URL https://doi.org/10.1146/annurev-financial-111914-041825. 8

Tobias Kretschmer, Eugenio J. Miravete, and Jose C. Pernias. Competitive Pressure and the Adoption of Complementary Innovations. *American Economic Review*, 102(4):1540–1570, June 2012. URL https://ideas.repec.org/a/aea/aecrev/v102y2012i4p1540-70.html. 7

Ryan Lampe and Petra Moser. Patent pools and innovation in substitute technologiesevidence from the 19th-century sewing machine industry. *The RAND Journal of Economics*, 44(4):757–778, 2013. 8

Ryan Lampe and Petra Moser. Patent pools, competition, and innovationevidence from 20 us industries under the new deal. *The Journal of Law, Economics, and Organization*, 32(1):1–36, 2016. 8

Ruitian Lang. Try before you buy: A theory of dynamic information acquisition. *Journal of Economic Theory*, 183:1057–1093, 2019. 5

Igor Letina. The road not taken: competition and the R&D portfolio. *RAND Journal of Economics*, 47(2):433–460, May 2016. URL https://ideas.repec.org/a/bla/randje/v47y2016i2p433-460.html. 6

Ángel L. López and Xavier Vives. Overlapping Ownership, R&D Spillovers, and Antitrust Policy. Technical report, 2016. 5, 10

Glenn C. Loury. Market Structure and Innovation. *The Quarterly Journal of Economics*, 93(3): 395–410, 1979. URL https://ideas.repec.org/a/oup/qjecon/v93y1979i3p395-410.html. 2

Guillermo Marshall and Álvaro Parra. Innovation and competition: The role of the product market. *International Journal of Industrial Organization*, 65:221 – 247, 2019. ISSN 0167-7187. doi: https://doi.org/10.1016/j.ijindorg.2019.04.001. URL http://www.sciencedirect.com/science/article/pii/S0167718719300207. 6

Stephen Martin. Endogenous firm efficiency in a cournot principal-agent model. *Journal of Economic Theory*, 59(2):445–450, 1993. URL https://EconPapers.repec.org/RePEc:eee:jetheo:v:59:y:1993:i:2:p:445-450. 2

Paul Milgrom and John Roberts. Rationalizability, learning, and equilibrium in games with strategic complementarities. *Econometrica*, 58(6):1255–77, November 1990. URL http://ideas.repec.org/a/ecm/emetrp/v58y1990i6p1255-77.html. 10

Giuseppe Moscarini and Lones Smith. The optimal level of experimentation. *Econometrica*, 69(6): 1629–1644, 2001. 5

Stephen Nickell. Competition and corporate performance. *Journal of Political Economy*, 104(4): 724–46, 1996. URL https://EconPapers.repec.org/RePEc:ucp:jpolec:v:104:y:1996:i:4:p:724-46. 7

Bernt Øksendal. *Stochastic differential equations: an introduction with applications*. Springer Science & Business Media, 2013. 16, 34

Michael E Porter. The competitive advantage of nations. *Competitive Intelligence Review*, 1(1):14–14, 1990. 2, 5

Jennifer F. Reinganum. The timing of innovation: Research, development, and diffusion. In R. Schmalensee and R. Willig, editors, *Handbook of Industrial Organization*, volume 1 of *Handbook of Industrial Organization*, chapter 14, pages 849–908. Elsevier, 1989. URL https://ideas.repec.org/h/eee/indchp/1-14.html. 5, 7

Kevin Roberts and Martin L Weitzman. Funding criteria for research, development, and exploration projects. *Econometrica: Journal of the Econometric Society*, pages 1261–1288, 1981. 5

Klaus M Schmidt. Managerial incentives and product market competition. *Review of Economic Studies*, 64(2):191–213, April 1997. URL http://ideas.repec.org/a/bla/restud/v64y1997i2p191-213.html. 2

Armin Schmutzler. Is Competition Good for Innovation? A Simple Approach to an Unresolved Question. *Foundations and Trends(R) in Microeconomics*, 5(6):355–428, August 2010. doi: 10.1561/0700000035. URL https://ideas.repec.org/a/now/fntmic/0700000035.html. 3

Armin Schmutzler. Competition and investment A unified approach. *International Journal of Industrial Organization*, 31(5):477–487, 2013. doi: 10.1016/j.ijindorg.2013.0. URL https://ideas.repec.org/a/eee/indorg/v31y2013i5p477-487.html. 6, 9

J.A. Schumpeter. *Capitalism, Socialism and Democracy.* Harper & Brothers, 1942. ISBN 9781134841509. URL https://books.google.cl/books?id=ytrqJswoRCoC. 2, 5

D. Siegmund. *Sequential Analysis: Tests and Confidence Intervals.* Springer-Verlag, New York, NY, 1985. 4

James E. Smith and Canan Ulu. Risk aversion, information acquisition, and technology adoption. *Operations Research*, 65(4):1011–1028, 2017. doi: 10.1287/opre.2017.1601. 5

Canan Ulu and James E. Smith. Uncertainty, information acquisition, and technology adoption. *Operations Research*, 57(3):740–752, 2009. doi: 10.1287/opre.1080.0611. 5

Xavier Vives. Innovation and competitive pressure. *Journal of Industrial Economics*, 56(3):419–469, December 2008. URL http://ideas.repec.org/a/bla/jindec/v56y2008i3p419-469.html. 5

S. Wager and K. Xu. Weak signal asymptotics for sequentially randomized experiments. Working paper, Stanford Graduate School of Business, 2023. URL https://arxiv.org/pdf/2101.09855.pdf. 5

A. Wald. Sequential tests of statistical hypotheses. *Ann. Math. Stat.*, 16(2):117–186, 1945. 2, 4

A. Wald and J. Wolfowitz. Optimum character of the sequential probability ratio test. *Ann. Math. Stat.*, 19(3):326–339, 1948. 4

A Proofs One Innovator Case

Proof of Lemma 1:

(a). If $\min\{\pi^{iG}(1, d^{-i}; \mu), \pi^{iB}(1, d^{-i}; \mu)\} \geq \pi^{i}(0, d^{-i}; \mu)$, then it follows from the definition of $V(\delta)$ that for any $\delta \in [0,1]$ $V(\delta) = (\pi^{i\theta}(1, d^{-i}; \mu) - (\pi(0, d^{-i}; \mu) - c))$. As a result, firm i's optimal stopping problem in (4) satisfies

$$\begin{aligned}
\mathcal{V}(\delta) = \sup_{\tau\in\mathbb{T}} \mathbb{E}_\delta \left[e^{-r\,\tau}\, V(\delta_\tau)\right] &= \sup_{\tau\in\mathbb{T}} \mathbb{E}_\delta \Big[e^{-r\,\tau} \left(\pi^{i\theta}(1, d^{-i}; \mu) - (\pi(0, d^{-i}; \mu) - c)\right)\Big] \\
&\leq \sup_{\tau\in\mathbb{T}} \mathbb{E}_\delta \Big[\pi^{i\theta}(1, d^{-i}; \mu) - (\pi(0, d^{-i}; \mu) - c)\big] \\
&= \mathbb{E}_\delta[\pi^{i\theta}(1, d^{-i}; \mu)] - (\pi(0, d^{-i}; \mu) - c),
\end{aligned}$$

where the last equality follows from the optional stopping theorem and the fact that δ_t is a bounded continuous martingale. Since $\mathbb{E}_\delta[\pi^{i\theta}(1, d^{-i}; \mu)] - (\pi(0, d^{-i}; \mu) - c) \geq 0$, we conclude that it is optimal for the firm to implement its idea immediately, i.e., $\tau^\star = 0$ and $d^\star = 1$.

(b). The proof follows trivially by noticing that the maximum expected payoff that the firm could get by implementing the idea equals $\max_{\delta\in(0,1)} \mathbb{E}_\delta[\pi^{i\theta}(1, d^{-i}; \mu)] - (\pi(0, d^{-i}; \mu) - c) < 0$. □

Proof of Theorem 1: For an $f \in \widehat{\mathcal{C}}^2$ that solves (QVI) we have

$$\begin{aligned}
e^{-r\,\tau}\, f(\delta_\tau) &= f(\delta) + \int_0^\tau e^{-r\,t}\, \mathcal{H}f(\delta_t)\,\mathrm{d}t + \int_0^\tau e^{-r\,t} \sigma\, \delta_t\,(1-\delta_t)\, f'(\delta_t)\,\mathrm{d}B_t \\
&\leq f(\delta) + \int_0^\tau e^{-r\,t}\, \sigma\, \delta_t\,(1-\delta_t)\, f'(\delta_t)\,\mathrm{d}B_t,
\end{aligned}$$

where the equality follows from integration-by-parts and Itô's lemma and the inequality follows from the fact that $\mathcal{H}f(\delta) \leq 0$ (second QVI condition). Taking expectation and canceling the stochastic integral, we get $\mathbb{E}[e^{-r\,\tau}\, f(\delta_\tau)] \leq f(\delta)$. From the first QVI condition it follows that $\mathbb{E}[e^{-r\,\tau}\, V(\delta_\tau)] \leq \mathbb{E}[e^{-r\,\tau}\, f(\delta_\tau)] \leq f(\delta)$. Taking the supreme over all stopping times $\tau \geq 0$, we conclude that $f(\delta) \geq \mathcal{V}(\delta)$. Finally, all the inequalities above become equalities for the QVI-control associated to f. This follows from Dynkin's formula and the fact that the QVI-control is the first exit time from the continuation region $\mathcal{C}$. □

Proof of Proposition 1: Let $\mathcal{V}(\delta)$ be the function defined in (11). We will show that $\mathcal{V}(\delta)$ satisfies the (QVI) optimality conditions and so by Theorem 1 it is equal to the firm's optimal expected payoff in (4). To this end, note that $\mathcal{V}(\delta) \in \widehat{\mathcal{C}}^2$, which follows from the smooth-pasting and value-matching conditions. Also, by (9) and the fact that $V(\delta)$ is piece-wise linear, we have that

$$(\mathcal{H}\mathcal{V})(\delta) = \begin{cases} -r\,V(\delta) & \text{if} \quad 0 \le \delta < \underline{\delta}, \\ \\ 0 & \text{if} \quad \underline{\delta} < \delta < \bar{\delta}, \\ \\ -r\,V(\delta) & \text{if} \quad \bar{\delta} < \delta \le 1. \end{cases}$$

From this, and the definition of $\mathcal{V}(\delta)$, it follows that $(\mathcal{H}\mathcal{V})(\delta) \le 0$ and $\big(\mathcal{V}(\delta) - V(\delta)\big)\,(\mathcal{H}\mathcal{V})(\delta) = 0$ for all $\delta \in [0,1] \setminus \{\underline{\delta}, \bar{\delta}\}$. Thus, $\mathcal{V}(\delta)$ satisfies the second and third (QVI) condition.

Next, we show the existence and uniqueness of a function $\mathcal{V}(\delta)$ satisfying the condition in the proposition. To simplify the notation let us define an auxiliary family of functions $\{\widehat{\mathcal{V}}(\delta;\bar{\delta})\colon \delta \in (0,1)\}$ parameterized by $\bar{\delta} \in (0,1)$ such that

$$\widehat{\mathcal{V}}(\delta;\bar{\delta}) = \ \beta_0(\bar{\delta})\,F(\delta) + \beta_1(\bar{\delta})\,F(1-\delta) \quad \text{if} \qquad 0 < \delta < \bar{\delta},$$

where the constants $\beta_0(\bar{\delta})$ and $\beta_1(\bar{\delta})$ are chosen so that $\widehat{\mathcal{V}}(\delta,\bar{\delta})$ is continuously differentiable at $\delta = \bar{\delta}$. To find $\beta_0(\bar{\delta})$ and $\beta_1(\bar{\delta})$, we impose value-matching and smooth-pasting conditions at $\delta = \bar{\delta}$:

$$\beta_0(\bar{\delta})\,F(\bar{\delta}) + \beta_1(\bar{\delta})\,F(1-\bar{\delta}) = 1 \qquad \text{and} \qquad \beta_0(\bar{\delta})\,F\delta(\bar{\delta}) + \beta_1(\bar{\delta})\,F_\delta(1-\bar{\delta}) = 0.$$

Using the fact that $F_\delta(\delta) = F(\delta)\dfrac{(1-\gamma-\delta)}{\delta\,(1-\delta)}$ and $F_\delta(1-\delta) = F(1-\delta)\dfrac{(\gamma-\delta)}{\delta\,(1-\delta)}$, we get that

$$\beta_0(\bar{\delta}) = \frac{(\gamma-\bar{\delta})}{(2\,\gamma-1)\,F(\bar{\delta})} \qquad \text{and} \qquad \beta_1(\bar{\delta}) = \frac{(\gamma+\bar{\delta}-1)}{(2\,\gamma-1)\,F(1-\bar{\delta})}.$$

Since $\gamma > 1$ it follows that $\beta_0(\bar{\delta})$ and $\beta_1(\bar{\delta})$ are both positive for $\bar{\delta} \in (0,1)$. Furthermore, $\beta_0(\bar{\delta}) \uparrow \infty$ as $\bar{\delta} \uparrow 1$ and $\beta_1(\bar{\delta}) \uparrow \infty$ as $\bar{\delta} \downarrow 0$. It follows that

$$\widehat{\mathcal{V}}(\delta;\bar{\delta}) = \begin{cases} \frac{(\gamma-\bar{\delta})}{(2\,\gamma-1)}\,\frac{F(\delta)}{F(\bar{\delta})} + \frac{(\gamma+\bar{\delta}-1)}{(2\,\gamma-1)}\,\frac{F(1-\delta)}{F(1-\bar{\delta})} & \text{if} \quad 0 < \delta < \bar{\delta}, \\ 1 & \text{if} \quad \bar{\delta} \le \delta \le 1. \end{cases} \tag{24}$$

By construction the function $\widehat{\mathcal{V}}(\delta;\bar{\delta})$ is continuously differentiable in $(0,1)$. Furthermore, in the region $\delta \in (0,\bar{\delta})$ it is also decreasing and strictly convex. To see this, note that in this region $\widehat{\mathcal{V}}(\delta;\bar{\delta})$ satisfies the differential equation (7) and so

$$\frac{(\sigma\delta(1-\delta))^2}{2}\widehat{\mathcal{V}}''(\delta;\bar{\delta}) - r\,\widehat{\mathcal{V}}(\delta;\bar{\delta}) = 0 \quad \Longrightarrow \quad \begin{aligned}\widehat{\mathcal{V}}''(\delta;\bar{\delta}) =& \frac{2}{(\sigma\delta(1-\delta))^2}\left(r\widehat{\mathcal{V}}(\delta;\bar{\delta})\right)\\ \geq& \frac{2}{(\sigma\delta(1-\delta))^2} > 0.\end{aligned}$$

This proves that it is strictly convex. In addition, from the smooth-pasting condition $\widehat{\mathcal{V}}'(\bar{\delta};\bar{\delta}) = 0$. As $\widehat{\mathcal{V}}'(\delta;\bar{\delta})$ increases with δ, we get that $\widehat{\mathcal{V}}'(\delta;\bar{\delta}) < 0$ in the region $\delta \in (0,\bar{\delta})$ proving that it is decreasing.

To complete the proof, we will show that there exists a value of

$$\bar{\delta} > \hat{\delta} := (\pi^{iG}(1,d^{-i};\mu) - \pi^{i}(0,d^{-i};\mu))/(\pi^{iG}(1,d^{-i};\mu) - \pi^{iB}(1,d^{-i};\mu)),$$

such that the associated function $\widehat{\mathcal{V}}(\delta;\bar{\delta})$ satisfies value-matching and smooth-pasting conditions with the function $(1-\delta)\,\pi^{iG}(1,d^{-i};\mu) + \delta\,\pi^{iB}(1,d^{-i};\mu)$ at some $\underline{\delta} < \hat{\delta}$.

The argument combines the following facts:

i) The function $\widehat{\mathcal{V}}(\delta;\bar{\delta})$ is monotonically decreasing and strictly convex in $(0,\bar{\delta}]$ as argued above.

ii) The function $V(\delta)$ is piece-wise linear in (0,1).

iii) $\widehat{\mathcal{V}}(\delta;\bar{\delta})$ is monotonic in $\bar{\delta}$, that is, $\widehat{\mathcal{V}}(\delta;\bar{\delta}_1) \leq \widehat{\mathcal{V}}(\delta;\bar{\delta}_2)$ for $\bar{\delta}_1 \leq \bar{\delta}_2$.

iv) For all $\bar{\delta} \in (0,1)$ we have that $\widehat{\mathcal{V}}(\delta;\bar{\delta}) \uparrow \infty$ as $\delta \downarrow 0$.

v) For $\bar{\delta}$ sufficiently large $\widehat{\mathcal{V}}(\delta;\bar{\delta}) > V(\delta)$ for all $\delta \in (0,\bar{\delta})$.

Point (iii) follows from noticing that in (A) the first-factor numerator is null when $\delta = \bar{\delta}$ and decreases with $\bar{\delta}$ as $F'(\delta) < 0$, then the numerator is negative for all $\delta < \bar{\delta}$. The denominator is always positive. Finally, the second factor is negative by $\gamma > 1$.

$$\frac{\partial\widehat{\mathcal{V}}(\delta;\bar{\delta})}{\partial\bar{\delta}} = \frac{1}{2\gamma-1}\left[\frac{F(1-\delta)F(\bar{\delta}) - F(\delta)F(1-\bar{\delta})}{F(1-\bar{\delta})F(\bar{\delta})}\right]\frac{\gamma(1-\gamma)}{\bar{\delta}(1-\bar{\delta})} > 0. \tag{25}$$

Point (iv) follows from noticing that $F(0) \uparrow \infty$ as $\delta \downarrow 0$. Finally, (v) follows the fact that $\beta_0(\bar{\delta})$ grows unboundedly as $\bar{\delta} \uparrow 1$.

Combining points (i) and (iv), it follows that if $\bar{\delta} \leq \hat{\delta}$, the function $\widehat{\mathcal{V}}(\delta;\bar{\delta})$ will intersect $V(\delta)$ in a non-smooth way in the region $(0,\bar{\delta})$. Thus, smooth-pasting can only be achieved if $\bar{\delta} > \hat{\delta}$. On the other hand, by point (v) for δ sufficiently large, the function $\widehat{\mathcal{V}}(\delta;\bar{\delta})$ never intersects $V(\delta)$ in $(0,\bar{\delta})$ and so again there is trivially no smooth-pasting in this region. Thus, by the continuity $\widehat{\mathcal{V}}(\delta;\bar{\delta})$ on δ and points (i) and (ii) there exists a $\bar{\delta}$ such that $\widehat{\mathcal{V}}(\delta;\bar{\delta})$ intersects smoothly $V(\delta)$ in the region $(0,\bar{\delta})$. Finally, by point (iii) there is a unique $\bar{\delta} \in (\hat{\delta},1)$ for which $\widehat{\mathcal{V}}(\delta;\bar{\delta})$ satisfies the smooth-pasting condition. □

Proof of Proposition 2: To derive the moment generating function $\mathbb{E}_\delta[e^{s\,\tau}]$ of τ, let us consider a function $f(\delta)$ such that $f(\underline{\delta}) = f(\bar{\delta}) = 1$ and

$$\frac{1}{2}\,\sigma^2\,\delta^2\,(1-\delta)^2\,f''(\delta) + s\,f(\delta) = 0 \quad \text{for all } \delta \in [\underline{\delta},\bar{\delta}].$$

For $s < \sigma^2/8$, the solution to this ODE is given by $f(\delta) = K_0\,F(\delta) + K_1\,F(1-\delta)$ for two constants of integration K_0 and K_1, where

$$F(\delta) = \frac{(1-\delta)^{\eta(s)}}{\delta^{\eta(s)-1}} \qquad \text{with } \eta(s) = \frac{1+\sqrt{1-8\,s/\sigma^2}}{2}.$$

We find the values of K_0 and K_1 imposing the boundary conditions $f(\underline{\delta}) = f(\bar{\delta}) = 1$. It follows that

$$f(\delta) = \frac{(F(1-\bar{\delta}) - F(1-\underline{\delta}))\,F(\delta) + (F(\underline{\delta}) - F(\bar{\delta}))\,F(1-\delta)}{F(\underline{\delta})\,F(1-\bar{\delta}) - F(\bar{\delta})\,F(1-\underline{\delta})}.$$

From Dynkin's formula (see Øksendal, 2013) we get

$$\mathbb{E}_\delta[e^{s\tau}\,f(\delta_\tau)] = f(\delta) + \mathbb{E}_\delta\left[\int_0^\tau \left(\frac{1}{2}\,\sigma^2\,\delta^2\,(1-\delta)^2\,f''(\delta) + s\,f(\delta)\right) e^{s\,t}\,\mathrm{d}t\right] = f(\delta).$$

But since $f(\underline{\delta}) = f(\bar{\delta}) = 1$ we have that $\mathbb{E}_\delta[e^{s\tau}\,f(\delta_\tau)] = \mathbb{E}_\delta[e^{s\,\tau}]$. We conclude that

$$\mathbb{E}_\delta[e^{s\,\tau}] = \frac{(F(1-\bar{\delta}) - F(1-\underline{\delta}))\,F(\delta) + (F(\underline{\delta}) - F(\bar{\delta}))\,F(1-\delta)}{F(\underline{\delta})\,F(1-\bar{\delta}) - F(\bar{\delta})\,F(1-\underline{\delta})}.$$

To compute the expected duration of due diligence, $\mathbb{E}_\delta[\tau]$, we can either evaluate the derivative of $\mathbb{E}_\delta[e^{s\,\tau}]$ with respect to s at $s = 0$. Alternatively, consider a function $g(\delta)$ such that

$$\frac{1}{2}\,\sigma^2\,\delta^2\,(1-\delta)^2\,g''(\delta) = 1 \quad \text{for all } \delta \in [\underline{\delta},\bar{\delta}].$$

One particular solution is given by

$$g(\delta) = \frac{2\,(1-2\delta)}{\sigma^2}\,\ln\left(\frac{1-\delta}{\delta}\right).$$

Then, it follows that

$$\mathbb{E}_\delta[g(\delta_\tau)] = g(\delta) + \mathbb{E}_\delta\left[\int_0^\tau \frac{1}{2}\,\sigma^2\,\delta^2\,(1-\delta)^2\,g''(\delta)\,\mathrm{d}t\right] = g(\delta) + \mathbb{E}_\delta[\tau].$$

But since $\mathbb{E}_\delta[g(\delta_\tau)] = g(\underline{\delta})\,\mathbb{P}_\delta(\delta_\tau = \underline{\delta}) + g(\bar{\delta})\,\mathbb{P}_\delta(\delta_\tau = \bar{\delta})$, we conclude that

$$\mathbb{E}_\delta[\tau] = \left(\frac{\bar{\delta}-\delta}{\bar{\delta}-\underline{\delta}}\right)\,g(\underline{\delta}) + \left(\frac{\delta-\underline{\delta}}{\bar{\delta}-\underline{\delta}}\right)\,g(\bar{\delta}) - g(\delta)$$

.

Finally, we use a similar derivation to compute $\mathbb{P}_\delta(\delta_\tau = \underline{\delta})$ and $\mathbb{P}_\delta(\delta_\tau = \bar{\delta}) = 1 - \mathbb{P}_\delta(\delta_\tau = \underline{\delta})$. Let us define the function $h(\delta)$ such that $h(\underline{\delta}) = 1$, $h(\bar{\delta}) = 0$ and

$$\frac{1}{2}\,\sigma^2\,\delta^2\,(1-\delta)^2\,h''(\delta) = 0 \quad \text{for all } \delta \in [\underline{\delta}, \bar{\delta}].$$

It follows that $h(\delta) = (\bar{\delta}-\delta)/(\bar{\delta}-\underline{\delta})$. Then

$$\mathbb{P}_\delta(\delta_\tau = \underline{\delta}) = \mathbb{E}_\delta[\mathbb{I}(\delta_\tau = \underline{\delta})] = \mathbb{E}_\delta[h(\delta_\tau)] = h(\delta) + \mathbb{E}_\delta\left[\int_0^\tau \frac{1}{2}\,\sigma^2\,\delta^2\,(1-\delta)^2\,h''(\delta)\,\mathrm{d}t\right] = h(\delta) = \frac{\bar{\delta}-\delta}{\bar{\delta}-\underline{\delta}}. \ \square$$

Proof of Proposition 3 and Proposition 4: The function $\widehat{\mathcal{V}}(\delta;\bar{\delta}^\star)$ must satisfy smooth-pasting and value-matching conditions at $\underline{\delta}^\star$ for each $\bar{\delta}$ satisfying its corresponding smooth-pasting and value-matching conditions.

Recall that

$$\widehat{\mathcal{V}}(\delta;\bar{\delta}) = \begin{cases} \frac{(\gamma-\bar{\delta})}{(2\,\gamma-1)}\,\frac{F(\delta)}{F(\bar{\delta})} + \frac{(\gamma+\bar{\delta}-1)}{(2\,\gamma-1)}\,\frac{F(1-\delta)}{F(1-\bar{\delta})} & \text{if} \quad 0 < \delta < \bar{\delta} \\ 1 & \text{if} \quad \bar{\delta} \le \delta \le 1. \end{cases} \tag{26}$$

Thus, value matching and smooth-pasting at $\underline{\delta}^\star$ entails the following

$$\widehat{\mathcal{V}}(\underline{\delta}^\star;\bar{\delta}^\star)\pi^i(0,d^{-i};\mu) = \underline{\delta}^\star\pi^{iB}(1,d^{-i};\mu) + (1-\underline{\delta}^\star)\pi^{iG}(1,d^{-i};\mu)$$

$$\widehat{\mathcal{V}}_\delta(\underline{\delta}^\star;\bar{\delta}^\star)\pi^i(0,d^{-i};\mu) = \pi^{iB}(1,d^{-i};\mu) - \pi^{iG}(1,d^{-i};\mu)$$

Let $\triangle_\pi(\mu) = \frac{\pi^{iB}(1,d^{-i};\mu)-\pi^{iG}(1,d^{-i};\mu)}{\pi^i(0,d^{-i};\mu)}$. Totally differentiating both equations with respect to $\bar{\delta}$, $\underline{\delta}$, and $(\pi^{iB}(1,d^{-i};\mu), \pi^{iG}(1,d^{-i};\mu)$, we get that

$$\pi^i(0,d^{-i};\mu)\begin{pmatrix} \widehat{\mathcal{V}}_{\underline{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) - \triangle_\pi(\mu) & \widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) \\ \widehat{\mathcal{V}}_{\delta\underline{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) & \widehat{\mathcal{V}}_{\delta\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) \end{pmatrix} \times \begin{pmatrix} \underline{\delta}^\star_{\pi_E} & \underline{\delta}^\star_{\pi_D} & \underline{\delta}^\star_{\pi} \\ \bar{\delta}^\star_{\pi_E} & \bar{\delta}^\star_{\pi_D} & \bar{\delta}^\star_{\pi} \end{pmatrix} = \begin{pmatrix} 1-\underline{\delta}^\star & \underline{\delta}^\star & -\widehat{\mathcal{V}}(\underline{\delta}^\star;\bar{\delta}^\star) \\ -1 & 1 & -\widehat{\mathcal{V}}_\delta(\underline{\delta}^\star;\bar{\delta}^\star) \end{pmatrix}$$

The determinant of the matrix is given by

$$
\begin{aligned}
\det \widehat{\mathcal{V}} \equiv & \big((\widehat{\mathcal{V}}_{\underline{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star) - \triangle_\pi(\mu)) \widehat{\mathcal{V}}_{\underline{\delta}\bar{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star) - \widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star) \widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star) = \\
& - \widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star) \widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star) < 0
\end{aligned}
$$

where the inequality readily follows from the fact that smooth-pasting implies that $\widehat{\mathcal{V}}_\delta(\underline{\delta}^\star; \bar{\delta}^\star) = \triangle_\pi(\mu)$, $\widehat{\mathcal{V}}(\delta; \bar{\delta}^\star)$ is decreasing and strictly convex in δ and non-decreasing in $\bar{\delta}$ for any given δ, and

$$
\frac{\partial \widehat{\mathcal{V}}(\underline{\delta}, \bar{\delta})}{\partial \underline{\delta}} = \frac{1}{2\gamma - 1} \frac{1}{\underline{\delta}(1-\underline{\delta})} \left((\gamma - \bar{\delta})(1 - \gamma - \underline{\delta}) \frac{F(\delta)}{F(\bar{\delta})} + (\gamma + \bar{\delta} - 1)(\gamma - \underline{\delta}) \frac{F(1-\delta)}{F(1-\bar{\delta})} \right) < 0,
$$

$$
\frac{\partial^2 \widehat{\mathcal{V}}(\underline{\delta}, \bar{\delta})}{\partial \underline{\delta} \partial \underline{\delta}} = -\frac{\gamma(1-\gamma)}{2\gamma - 1} \frac{1}{(\underline{\delta}(1-\underline{\delta}))^2} \left((\gamma - \bar{\delta}) \frac{F(\delta)}{F(\bar{\delta})} + (\gamma + \bar{\delta} - 1) \frac{F(1-\delta)}{F(1-\bar{\delta})} \right) > 0,
$$

$$
\frac{\partial \widehat{\mathcal{V}}(\delta; \bar{\delta})}{\partial \bar{\delta}} = \frac{1}{2\gamma - 1} \left(\frac{F(1-\delta)}{F(1-\bar{\delta})} - \frac{F(\delta)}{F(\bar{\delta})} \right) \frac{\gamma(1-\gamma)}{\bar{\delta}(1-\bar{\delta})} > 0,
$$

and

$$
\frac{\partial^2 \widehat{\mathcal{V}}(\underline{\delta}, \bar{\delta})}{\partial \underline{\delta} \partial \bar{\delta}} = \frac{\gamma(1-\gamma)}{2\gamma - 1} \frac{1}{\underline{\delta}(1-\underline{\delta})\bar{\delta}(1-\bar{\delta})} \left((\gamma - \underline{\delta}) \frac{F(1-\underline{\delta})}{F(1-\bar{\delta})} + (\gamma + \underline{\delta} - 1) \frac{F(\underline{\delta})}{F(\bar{\delta})} \right) < 0.
$$

Thus,

$$
\begin{aligned}
\widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}; \bar{\delta}) + \widehat{\mathcal{V}}_{\underline{\delta}\bar{\delta}}(\underline{\delta}; \bar{\delta}) = & \frac{\gamma(1-\gamma)}{2\gamma - 1} \frac{1}{\underline{\delta}(1-\underline{\delta})} \left(\left(\frac{\gamma - \underline{\delta}}{\bar{\delta}(1-\bar{\delta})} - \frac{\gamma + \bar{\delta} - 1}{\underline{\delta}(1-\underline{\delta})} \right) \frac{F(1-\underline{\delta})}{F(1-\bar{\delta})} + \right. \\
& \left. \left(\frac{\gamma + \underline{\delta} - 1}{\bar{\delta}(1-\bar{\delta})} - \frac{\gamma - \bar{\delta}}{\underline{\delta}(1-\underline{\delta})} \right) \frac{F(\underline{\delta})}{F(\bar{\delta})} \right).
\end{aligned}
$$

Next, Cramer's rule implies that

$$
\underline{\delta}^\star_{\pi_D} = \frac{\underline{\delta}^\star \widehat{\mathcal{V}}_{\underline{\delta}\bar{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star) - \widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star)}{\det \widehat{\mathcal{V}}} > 0,
$$

$$
\underline{\delta}^\star_{\pi_E} = \frac{(1 - \underline{\delta}^\star) \widehat{\mathcal{V}}_{\underline{\delta}\bar{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star) + \widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star)}{\det \widehat{\mathcal{V}}} > 0,
$$

$$
\bar{\delta}^\star_{\pi_D} = \frac{-\underline{\delta}^\star \widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star)}{\det \widehat{\mathcal{V}}} > 0,
$$

$$
\bar{\delta}^\star_{\pi_E} = \frac{-(1 - \underline{\delta}^\star) \widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star)}{\det \widehat{\mathcal{V}}} > 0,
$$

$$
\underline{\delta}^\star_{\pi} = \frac{-\widehat{\mathcal{V}}(\underline{\delta}^\star; \bar{\delta}^\star) \widehat{\mathcal{V}}_{\bar{\delta}\underline{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star) + \widehat{\mathcal{V}}_{\delta}(\underline{\delta}^\star; \bar{\delta}^\star) \widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star; \bar{\delta}^\star)}{\det \widehat{\mathcal{V}}} < 0
$$

and

$$\bar{\delta}^\star_\pi = \frac{\widehat{\mathcal{V}}(\underline{\delta}^\star;\bar{\delta}^\star)\widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star)}{\det \widehat{\mathcal{V}}} < 0,$$

Observe that if

$$\underline{\delta}^\star \left(\pi^{iB}_\mu - \frac{\pi^{iB}}{\pi^i}\pi^i_\mu\right) + (1-\underline{\delta}^\star)\left(\pi^{iG}_\mu - \frac{\pi^{iG}}{\pi^i}\pi^i_\mu\right) \leq 0,$$

then

$$\bar{\delta}^\star_\mu = \bar{\delta}^\star_{\pi_D}\pi^{iB}_\mu(1,d^{-i};\mu) + \bar{\delta}^\star_{\pi_E}\pi^{iG}_\mu(1,d^{-i};\mu) + \bar{\delta}^\star_\pi \pi^i_\mu(0,d^{-i};\mu) \leq 0,$$

$$\underline{\delta}^\star_\mu = \underline{\delta}^\star_{\pi_D}\pi^{iB}_\mu(1,d^{-i};\mu) + \underline{\delta}^\star_{\pi_E}\pi^{iG}_\mu(1,d^{-i};\mu) + \underline{\delta}^\star_\pi \pi^i_\mu(0,d^{-i};\mu) < 0,$$

while if

$$\underline{\delta}^\star \left(\pi^{iB}_\mu - \frac{\pi^{iB}}{\pi^i}\pi^i_\mu\right) + (1-\underline{\delta}^\star)\left(\pi^{iG}_\mu - \frac{\pi^{iG}}{\pi^i}\pi^i_\mu\right) > 0,$$

$\bar{\delta}^\star_\mu > 0$ and $\underline{\delta}^\star_\mu > 0$ whenever $\underline{\delta}^\star \geq 1/2$ or $\underline{\delta}^\star < 1/2$ and

$$\underline{\delta}^\star\left((\gamma-1)\frac{F(1-\underline{\delta}^\star)}{F(1-\underline{\delta}^\star)} + \gamma\frac{F(\underline{\delta}^\star)}{F(\underline{\delta}^\star)}\right)\left(\pi^{iB}_\mu - \frac{\pi^{iB}}{\pi^i}\pi^i_\mu\right) +$$
$$(1-\underline{\delta}^\star)\left((\gamma-2\underline{\delta}^\star)\frac{F(1-\underline{\delta}^\star)}{F(1-\underline{\delta}^\star)} + (\gamma+2\underline{\delta}^\star-1)\frac{F(\underline{\delta}^\star)}{F(\underline{\delta}^\star)}\right)\left(\pi^{iG}_\mu - \frac{\pi^{iG}}{\pi^i}\pi^i_\mu\right) \geq 0.$$

Observe that

$$\bar{\delta}^\star_\mu - \underline{\delta}^\star_\mu = (\bar{\delta}^\star_{\pi_D} - \underline{\delta}^\star_{\pi_D})\pi^{iB}_\mu(1,d^{-i};\mu) + (\bar{\delta}^\star_{\pi_E} - \underline{\delta}^\star_{\pi_E})\pi^{iG}_\mu(1,d^{-i};\mu) + (\bar{\delta}^\star_\pi - \underline{\delta}^\star_\pi)\pi^i_\mu(0,d^{-i};\mu) \leq 0,$$

where

$$\bar{\delta}^\star_{\pi D} - \underline{\delta}^\star_{\pi D} = \frac{\widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) - \underline{\delta}^\star(\widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) + \widehat{\mathcal{V}}_{\underline{\delta}\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star))}{\det \widehat{\mathcal{V}}} < 0,$$

$$\bar{\delta}^\star_{\pi E} - \underline{\delta}^\star_{\pi E} = \frac{-(1-\underline{\delta}^\star)(\widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) + \widehat{\mathcal{V}}_{\underline{\delta}\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star)) - \widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star)}{\det \widehat{\mathcal{V}}} > 0 \text{ iff } \bar{\delta}^\star < 2\underline{\delta}^\star.$$

and

$$\bar{\delta}^\star_\pi - \underline{\delta}^\star_\pi = \frac{\widehat{\mathcal{V}}(\underline{\delta}^\star;\bar{\delta}^\star)(\widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) + \widehat{\mathcal{V}}_{\bar{\delta}\underline{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star)) - \widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star)\widehat{\mathcal{V}}_{\delta}(\underline{\delta}^\star;\bar{\delta}^\star)}{\det \widehat{\mathcal{V}}} < 0.$$

It readily follows from this that

$$\bar{\delta}^\star_\mu - \underline{\delta}^\star_\mu = -\frac{\widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) + \widehat{\mathcal{V}}_{\underline{\delta}\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star))}{\det \widehat{\mathcal{V}}}\left(\underline{\delta}^\star\left(\pi^{iB}_\mu - \frac{\pi^{iB}}{\pi^i}\pi^i_\mu\right) + (1-\underline{\delta}^\star)\left(\pi^{iG}_\mu - \frac{\pi^{iG}}{\pi^i}\pi^i_\mu\right)\right) +$$
$$\frac{\widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star)}{\det \widehat{\mathcal{V}}}\left(\pi^{iB}_\mu - \frac{\pi^{iB}}{\pi^i}\pi^i_\mu - \left(\pi^{iG}_\mu - \frac{\pi^{iG}}{\pi^i}\pi^i_\mu\right)\right).$$

Because of Assumption 3, $\widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) > 0$, $\det \widehat{\mathcal{V}} < 0$, the second term is positive.

Recall that

$$\mathbb{P}_{\bar{\delta}}(d^\star = 1) = \frac{\bar{\delta}^\star - \delta}{\bar{\delta}^\star - \underline{\delta}^\star}$$

and therefore

$$\frac{\partial \mathbb{P}_\delta(d^\star = 1)}{\partial \mu} = \bar{\delta}^\star_\mu \frac{\delta - \underline{\delta}^\star}{(\bar{\delta}^\star - \underline{\delta}^\star)^2} + \underline{\delta}^\star_\mu \frac{\bar{\delta}^\star - \delta}{(\bar{\delta}^\star - \underline{\delta}^\star)^2} \leq 0,$$

whenever

$$\underline{\delta}^\star \left(\pi^{iB}_\mu - \frac{\pi^{iB}}{\pi^i} \pi^i_\mu \right) + (1 - \underline{\delta}^\star) \left(\pi^{iG}_\mu - \frac{\pi^{iG}}{\pi^i} \pi^i_\mu \right) \leq 0.$$

and it is positive when the opposite holds and

Recall that

$$\mathbb{E}_\delta[\tau^\star] = \mathbb{P}_\delta(d^\star = 1)\, g(\underline{\delta}^\star) + \mathbb{P}_\delta(d^\star = 0)\, g(\bar{\delta}^\star) - g(\delta), \quad \text{where} \quad g(\delta) = \frac{2\,(1 - 2\delta)}{\sigma^2} \ln\left(\frac{1-\delta}{\delta}\right).$$

It readily follows from this that

$$\frac{\partial \mathbb{E}_\delta[\tau^\star]}{\partial \mu} = \frac{\partial \mathbb{P}_\delta(d^\star = 1)}{\partial \mu}(g(\underline{\delta}^\star) - g(\bar{\delta}^\star)) + \mathbb{P}_\delta(d^\star = 1)\, g'(\underline{\delta}^\star)\underline{\delta}^\star_\mu + \mathbb{P}_\delta(d^\star = 0)\, g'(\bar{\delta}^\star)\bar{\delta}^\star_\mu,$$

where

$$g'(\delta) = \frac{2}{\sigma^2}\left(-2 \ln \frac{1-\delta}{\delta} - \frac{1 - 2\delta}{\delta(1-\delta)} \right).$$

Thus, $g'(\delta) \leq 0$ for $\delta \leq 1/2$ and $g'(\delta) > 0$ otherwise and $g''(\delta) \geq 0$. In addition, $g(\underline{\delta}) - g(\bar{\delta}) \geq 0$ if $\bar{\delta} \leq 1/2$, $g(\underline{\delta}) - g(\bar{\delta}) < 0$ if $\underline{\delta} \geq 1/2$, and $g(\underline{\delta}) - g(\bar{\delta}) \lesseqgtr 0$ if $\underline{\delta} \leq 1/2 < \bar{\delta}$.

Observe that

$$\frac{\partial \mathbb{E}_\delta[\tau^\star]}{\partial \mu} = \mathbb{P}_\delta(d^\star = 0)\bar{\delta}^\star_\mu \left(g'(\bar{\delta}^\star) - \frac{g(\bar{\delta}^\star) - g(\underline{\delta}^\star)}{\bar{\delta}^\star - \underline{\delta}^\star} \right) + \mathbb{P}_\delta(d^\star = 1)\underline{\delta}^\star_\mu \left(g'(\underline{\delta}^\star) - \frac{g(\bar{\delta}^\star) - g(\underline{\delta}^\star)}{\bar{\delta}^\star - \underline{\delta}^\star} \right).$$

Because $g(\cdot)$ is strictly convex and $\bar{\delta}^\star > \underline{\delta}^\star$, the first term in parenthesis is positive and the second is negative, if $\underline{\delta}^\star_\mu \geq 0$, and $\bar{\delta}^\star_\mu \leq 0$, the expected time falls with μ. This happens whenever

$$\frac{\underline{\delta}^\star \pi^{iB}_\mu + (1 - \underline{\delta}^\star)\pi^{iG}_\mu}{\pi^i_\mu} < \frac{\underline{\delta}^\star \pi^{iB} + (1 - \underline{\delta}^\star)\pi^{iG}}{\pi^i}.$$

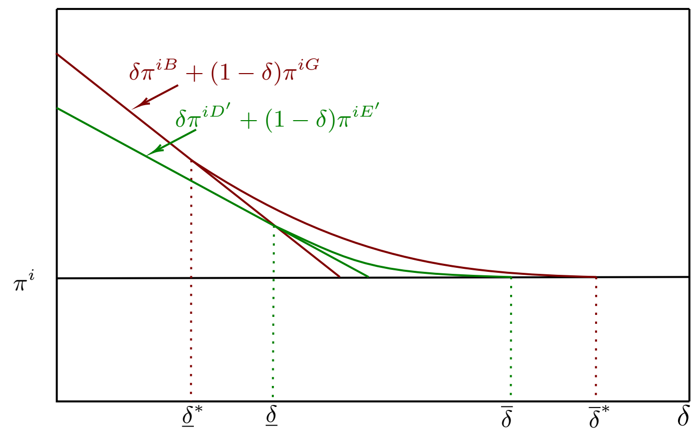

Figure 2: Comparative Statics

Proof of Proposition 3*:* Value matching and smooth-pasting at $\underline{\delta}^\star$ entail the following

$$\widehat{\mathcal{V}}(\underline{\delta}^\star;\bar{\delta}^\star)\mathcal{V}(\eta;\rho) = \delta^\star\pi^{iB}(1,d^{-i};\mu) + (1-\delta^\star)\pi^{iG}(1,d^{-i};\mu)$$

$$\widehat{\mathcal{V}}_\delta(\underline{\delta}^\star;\bar{\delta}^\star)\mathcal{V}(\eta;\rho) = \pi^{iB}(1,d^{-i};\mu) - \pi^{iG}(1,d^{-i};\mu)$$

Totally differentiating both equations with respect to $\bar{\delta}$, $\underline{\delta}$, and γ, we get that

$$\begin{pmatrix} \mathcal{V}(\eta;\rho)\widehat{\mathcal{V}}_{\underline{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) - \triangle_\pi(\mu) & \mathcal{V}(\eta;\rho)\widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) \\ \mathcal{V}(\eta;\rho)\widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) & \mathcal{V}(\eta;\rho)\widehat{\mathcal{V}}_{\underline{\delta}\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) \end{pmatrix} \times \begin{pmatrix} \underline{\delta}^\star_{\gamma(\rho)} \\ \underline{\delta}^\star_{\gamma(\rho)} \end{pmatrix} = \begin{pmatrix} -\mathcal{V}_{\gamma(\rho)}(\eta;\rho)\widehat{\mathcal{V}}(\underline{\delta}^\star;\bar{\delta}^\star) \\ -\mathcal{V}_{\gamma(\rho)}(\eta;\rho)\widehat{\mathcal{V}}_\delta(\underline{\delta}^\star;\bar{\delta}^\star) \end{pmatrix}$$

where

$$\mathcal{V}_{\gamma(\rho)}(\eta;\rho) \geq 0 \text{ iff } \bar{\eta}^\star \leq 1/2 \text{ and negative oherwise}$$

and the sign of the first inequality follows from the following facts: i) $F(\eta)$ rises and $F(1-\eta)$ falls with γ whenever $\delta \leq 1/2$; ii) $\frac{\gamma(1-\gamma)}{2\gamma-1}$ falls with γ; iii) $\frac{F(1-\eta)}{F(1-\bar{\eta})}$ falls and $\frac{F(\eta)}{F(\bar{\eta})}$ rises with γ for all $\eta \leq \bar{\eta}$; and iv) $\frac{(\gamma-\bar{\eta})}{(2\gamma-1)}$ rises and $\frac{(\gamma+\bar{\eta}-1)}{(2\gamma-1)}$ falls with γ whenever $\bar{\eta} \geq 1/2$.

Next, Cramer's rule implies that

$$\underline{\delta}^\star_{\gamma(\rho)} = \frac{-\widehat{\mathcal{V}}_{\underline{\delta}\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star) + \widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^\star;\bar{\delta}^\star)}{\det\widehat{\mathcal{V}}}\mathcal{V}_{\gamma(\rho)}(\eta;\rho)\mathcal{V}(\eta;\rho)\widehat{\mathcal{V}}(\underline{\delta}^\star;\bar{\delta}^\star) \leq 0 \,\text{iff}\, \bar{\eta}^\star \leq 1/2,$$

and

$$\bar{\delta}^\star_{\gamma(\rho)} = \frac{\widehat{\mathcal{V}}_{\underline{\delta}\delta}(\underline{\delta}^\star;\bar{\delta}^\star)}{\det\widehat{\mathcal{V}}}\mathcal{V}_{\gamma(\rho)}(\eta;\rho)\mathcal{V}(\eta;\rho)\widehat{\mathcal{V}}(\underline{\delta}^\star;\bar{\delta}^\star) \leq 0 \,\text{iff}\, \bar{\eta}^\star \leq 1/2.$$

Thus,

$$\bar{\delta}^{\star}_{\gamma(\rho)} - \underline{\delta}^{\star}_{\gamma(\rho)} = \frac{\widehat{\mathcal{V}}_{\underline{\delta}\bar{\delta}}(\underline{\delta}^{\star};\bar{\delta}^{\star}) + \widehat{\mathcal{V}}_{\underline{\delta}\underline{\delta}}(\underline{\delta}^{\star};\bar{\delta}^{\star}) - \widehat{\mathcal{V}}_{\bar{\delta}}(\underline{\delta}^{\star};\bar{\delta}^{\star})}{\det \widehat{\mathcal{V}}} \mathcal{V}_{\gamma(\rho)}(\eta;\rho)\mathcal{V}(\eta;\rho)\widehat{\mathcal{V}}(\underline{\delta}^{\star};\bar{\delta}^{\star}) \geq 0 \text{ iff } \bar{\eta}^{\star} \leq 1/2.$$

□

PROOF OF PROPOSITION 7 AND PROPOSITION 8: The probability that the idea is implemented when $d^1 = 1$ is larger than that when $d^1 = 0$ whenever $(\eta - \underline{\eta}^{\star}(0))(\bar{\eta}^{\star}(1) - \bar{\eta}^{\star}(0)) > (\eta - \bar{\eta}^{\star}(0))(\underline{\eta}^{\star}(1) - \underline{\eta}^{\star}(0))$. □