Labor Market Power, Automation, and Generative AI

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Abstract

This paper argues that in labor markets where firms have market power automation targets low-rent jobs, increasing rents and amplifying wage losses from automation. This implies that more jobs are automated relative to those in a competitive labor market and those that maximize welfare. Conditional on a job not being automated, the assignment of workers to jobs complemented with generative AI is efficient. The allocative inefficiency due to too much automation implies lower wages and more displaced workers relative to a competitive labor market. Taxing technological capital can restore allocative efficiency in terms of automation but at the cost of lowering wages and inefficiently low adoption of generative AI. Automation may induce workers to overinvest in skills to avoid being displaced by automation.

JEL: J3, D2, J30

Key Words: Labor market power, automation, generative AI, wages, productivity, displaced workers, taxes.

1 Introduction

In Marx (1894), Marx argued that business owners invest in labor-saving machines when wages become too high, creating a "reserve army of labor" that results in an excess supply of labor and, consequently, lower wages. Simultaneously, more capital leads to expanded production, increasing the demand for labor. In his own words, "Capital works on both sides at the same time. If its accumulation, on the one hand, increases the demand for labour, it increases, on the other, the supply of labourers by the 'setting free' of them" (Marx (1894), sect. 3, last para.).

Bessen (2015) asserts that labor markets were not competitive during the times Marx considered when formulating his theories. Workers had fewer alternative sources of employment, a requisite for Marx's argument to be plausible. However, as technological change expanded economic growth and created new employment opportunities, even workers with modest skills, like spinners, saw their wages increase, and those with more specialized technical skills—such as weavers—benefited disproportionately.

Contemporary labor markets worldwide have evolved to resemble the ones that Marx had in mind. Firstly, labor market power has increased in most labor markets over the last two decades. For instance, Manning's (2021) review reports that firms wield considerable market power, even in online markets that, a priori, one would expect to be highly competitive. This power is exercised by employers, resulting in lower wages. Secondly, jobs are being automated and complemented with AI at an increasing rate. The former results in an increasing number of displaced workers while the latter in a larger labor productivity.[1] For instance, Acemoglu and Restrepo (2021) document that between 50% and 70% of changes in the US wage structure over the last four decades can be explained by the relative wage decline of workers specializing in routine tasks in labor markets experiencing rapid automation. Bessen, Goos, Salomons, and van den Berge (2020) show that automating firms experience faster employment and revenue growth than non-automating firms. However, at the time the automation events occur, there is a large drop in employment growth. Aghion, Antonin, Bunel, and Jaravel (2022) find that at the plant, firm, and industry level, automation positively affects employment, leads to higher profits, lower consumer prices, and higher sales, while it leaves wages, the labor share, and within-firm wage inequality unchanged. **?** creates indices of occupational exposure to robotics, software, and artificial intelligence. He finds

[1]The annual number of installations of industrial robots worldwide more than doubled from 2012 to 2019, reaching 373,240 by the end of that period. The operational global stock of robots increased from 1 million in 2009 to about 2.7 million in 2019.

that occupations highly exposed to previous automation technologies saw declines in employment and wages over the relevant periods. He also finds that, in contrast to software and robots, AI is directed at high-skilled tasks. He also estimates that AI will reduce 90:10 wage inequality, but will not affect the top 1%. Autor, Chin, Salomons, and Seegmiller (2022) develop text-based measures of occupational exposure to automation technologies and augmenting technologies because they argue that job titles capture the services rendered in an occupation (and not the tasks involved in rendering these services). Using US data for 1940–1980 and 1980–2018, they show that the share of employment and wage payments expanded in occupations exposed to augmenting innovations and contracted for those exposed to automation innovations.

This paper answers the following questions: When do firms with labor market power automate jobs? When do firms with market power assign workers to jobs that use generative AI? Does labor market power distort the productive and allocative efficiency of jobs? Does this distort workers' human capital investment decisions? Should economies tax technological capital?[2]

To answer these questions, we consider a model with n firms operating in a perfectly competitive product market and an imperfectly competitive labor market. Jobs can be automated –automation fully substitutes for human skills– or assigned to workers. There are two types of human-skill jobs: human-skill-only jobs–the only input is human skills– and human-skills-generative-AI jobs. Generative AI complements human skills and is based on new algorithm developments that allow computers and machines to learn, create hypotheses, and solve complex problems that used to be fully solved by human brains and with this technology are solved by human brains with the support and insights from modern algorithms. Low-skill workers have a comparative advantage in human-skill-only jobs and high-skill workers have a comparative advantage in human-skills-generative-AI jobs.

An example of automated jobs is Kiva Systems, acquired by Amazon in 2012, to automatize their warehouses. The Kiva system consists of a dispatch program overseeing goods flow in the warehouse and coordinating robots' shelf transport with human work. An example of a job that uses human skills and generative AI is Google's car which adapts in real-time to obstacles, such as cars, pedestrians, and road hazards, by braking, turning, and stopping. However, when the car's software determines that the operating environment differs from the programmed environment;

[2]There are a few article that study the interplay between technology and labor market imperfections, but these do not study how the presence of labor market power impacts the relationship between automation, wages, and productivity. The exception is Acemoglu and Restrepo (2024) who study a similar issue in a completely different setting where workers instead of firms have rents, which are exogenously given. They show that automation is concentrated in high-rent jobs, dissipating rents and amplifying wage losses from automation.

e.g., when it encounters an unexpected detour or a crossing guard instead of a traffic signal, the car requires its human operator to take control.

Workers are indexed according to their type or class. Workers from a higher class have a skill distribution that dominates in the first-order stochastic sense (FOSD) the skill distribution of workers from lower classes. Workers have an outside-option payoff –self-employment, home production, unemployment benefits, etc.– and preferences over firms. These are modeled as a random non-pecuniary additive benefit distributed according to a log-concave distribution. The skill and non-pecuniary benefit distributions are common knowledge. Workers' preference heterogeneity across firms gives rise to labor market power. This places our model within the literature on market power due to horizontal job differentiation.

The timing of decisions is as follows. First, for each worker's class, firms simultaneously choose whether or not to automate jobs. For jobs that are not automated, firms simultaneously offer wages conditional on the worker's class. After that, the non-pecuniary benefits and skills are realized and become common knowledge. Then, individuals decide whether to work for one of the n firms or to take the outside-option payoff. After that, firms choose whether or not to assign a worker to a job with generative AI.

There is a unique symmetric sub-game perfect equilibrium for each workers' class. In the last stage, a firm that did not automate the job adopts generative AI when the rent from producing with human skills and generative AI is larger than that from a human-skill-only job. Because the wage is already set, the rent difference is independent of the wage. Thus, a human-skill-only job is adopted when the marginal product of labor is larger than that minus technological costs for a job with human skills and generative AI. This occurs when the realized skill is smaller than a given skill threshold since low-skill workers have a comparative advantage in human-skill-only jobs and high-skill workers have a comparative advantage when generative AI is adopted.

For each worker's class, there is a unique equilibrium wage. The equilibrium wage is lower than workers' marginal product of labor and the mark-down is equal to the inverse of the labor-supply elasticity. The pass-through from the marginal product of labor and the outside-option payoff to wages is positive and lower than 1.

Workers choose between the firm that offers the highest payoff (wage plus the non-pecuniary benefit) and the outside-option payoff. Because non-pecuniary benefits are random, from the firms' point of view, the labor supply is the probability that a worker chooses the corresponding firm. Thus, the labor supply is the cumulative distribution function (CDF) of the first-highest statistics provided that the paid-employment is preferred to outside-option payoff. Uniqueness is derived from the log-concave distribution of non-pecuniary benefits, which causes the CDF of the second-highest statistic to increase in the sense of first-order stochastic dominance with the difference between the payoff of self-employment and that of paid employment.

Labor market power is measured by Lerner's index which, in equilibrium, is the inverse of the labor supply elasticity. This can be decomposed into two effects: the market exclusion effect –equivalent to the exclusion effect in the market for goods– and the competition effect.

The market exclusion effect measures the case when valuations for all other firms are below a given threshold and thereby the firm acts as a monopsony. In this case, lowering the wage by a small amount will exclude a few individuals from paid employment. The competition effect –up to the adjustment that the marginal individual's valuation for paid employment is given by the difference between the outside-option payoff and the wage– measures the density of a firm's marginal workers; that is, those who are indifferent between the corresponding firm and the best outside option for them.

In the first stage, a firm automates a job when the rent from automation –the productivity of automation minus the technological costs– is higher than the expected rent from hiring a worker –the expected markdown times the labor supply minus the technological cost for those stated in which generative AI is adopted–. Because the expected marginal product of labor increases with a first-order stochastic improvement in the skill distribution, and the pass-through from productivity to wages is positive but lower than 1, the expected rent from human-skill jobs increases with the workers' class. Thus, lower-class workers' jobs are prone to be automated and, as a result, lower-class workers are displaced. For non-displaced workers, those with higher realized skills are assigned to a job complemented with generative AI, and those with lower skill realizations are assigned to a job where human skills are the only input.

In equilibrium, there is a productive inefficiency since some jobs are automated despite displaced workers being more productive than automation. Thus, there is "too much" automation when there is labor market power. There is also an allocation inefficiency since total welfare is larger when jobs are assigned to workers than when jobs are automated. This happens because firms make these decisions based on differences in rents instead of differences in productivity/welfare. These novel inefficiencies have three consequences: inefficiently displaced workers, loss of non-pecuniary benefits, and lower average productivity.

Because market power results in an allocative inefficiency, our setting lends itself to evaluating the common claim that technological capital should be taxed since automation results in displaced workers with the corresponding welfare loss. We show that in a symmetric equilibrium, there is a tax rate that fully restores allocative efficiency in terms of automation. However, this comes at the cost of creating an allocative inefficiency about generative AI adoption with the corresponding productivity loss and wage reduction for non-displaced workers. Thus, restoring allocative efficiency concerning automation might not be welfare-maximizing.

Last but not least, we show that automation may induce workers to overinvest in skills to avoid being displaced by it. This happens when either the skill requirement to escape automation is not too large or the outside-option payoff, such as the one from home production and self-employment, is not too large or both. Otherwise, workers prefer not to invest in skills, to be displaced, and to take the outside-option payoff. This results in an extreme form of underinvestment.

The rest of the paper is structured as follows. In Section 2, we present the model. In the next one, Section 3, we derive the sub-game perfect equilibrium. Then, in Section 4, we study taxing technological capital. In Section 5, we study how labor market power and allocative inefficiency affect individuals' incentives to invest in human capital. In Section 6, we provide concluding remarks.

2 The Model

2.1 Set-Up

Let's consider the following labor-market game. In the first period, firms decide whether or not to automatize the job. After that, if the job is not automatized, wages are simultaneously chosen. In the third period, individuals learn the non-pecuniary preferences and the wage for each firm and choose to supply their labor to the firm that offers the higher utility, provided that this is higher than the outside-option payoff. After that firms learn applicants' skill level and choose whether to assign the worker to a job that uses generative AI and human skills or to only human-skills jobs.

To keep the analysis simple, firms and individuals are risk-neutral and do not discount the future. Firms separate workers into different human capital or credential groups or classes, denoted by s, where s could be, for instance, college degree workers, high-school graduates, etc. Firms believe that workers belonging to class s have a skill cumulative distribution function $F(t|s)$, with full support $[0, \bar{t}]$ and density $f(t|s)$. The class to which a worker belongs and $f(t|s)$ are common knowledge. Workers know their skill level t. A worker who is not hired gets his outside option payoff, which yields a payoff b. A worker from skill class $s' > s$ has distribution $F(t|s')$ that dominates $F(t|s)$ in the sense of first-order stochastic dominance. Thus, $F_s(t|s) < 0$.

There are n firms horizontally differentiated from individuals' point of view, indexed by $j \in \{1, \dots, n\}$. Firms produce tradeable goods that are perfect substitutes, and so they trade in a perfectly competitive market at a price p, normalized to one.

We model such occupational differentiation by adopting a random-utility framework in the spirit of Perloff and Salop (1985). Let $\epsilon_l = (\epsilon_l^j, \dots, \epsilon_n^j)$ be the match-specific utility shock of individual l in each of the $j \in \{1, \dots, n\}$ possible firm/jobs. Thus, the utility of individual l in job j is given by: $w^j + \epsilon_l^j$. We assume that ϵ_l is i.i.d. across individuals, which reflects idiosyncratic tastes for different jobs, and for a given worker, it is also i.i.d. across jobs. These non-wage job characteristics might include hours of work, distance of the firm from the worker's home, the social environment in the workplace, etc. In the forthcoming analysis, we will suppress the index l. ϵ^j is distributed $G(\cdot)$ with compact and full support $[\underline{\epsilon}, \bar{\epsilon}] \subset \Re$, zero mean and twice differentiable density $g(\cdot)$. Firms choose wages simultaneously and do not discriminate among individuals of the same type or class. Thus, there is a labor market for each s-type worker. The next assumption is crucial for characterizing the labor-market equilibrium.

Assumption 1. *$g(\epsilon)$ is log-concave.*

Firms have access to a constant return to scale technology; i.e., the total output of a firm is equal to the sum of each job's output.[3]

Jobs are of three different types: automated jobs (a) that fully substitute for human skills; human-skills-only jobs (h) whose only input is human skills; and generative AI-human skills jobs (g) that use both human skills and generative AI. The type of job is denoted by $\tau \in \{a, g, h\}$.

The output in job τ in firm j when the worker's skill turns out to be t is given by

$$y^j(t; \tau) = \sigma^j(\tau) + \phi^j(\tau)t.$$

Let $t_g^j \equiv \frac{\sigma^j(a) - \sigma^j(g)}{\phi^j(g)}$ and $t_h^j \equiv \frac{\sigma^j(a) - \sigma^j(h)}{\phi^j(h)}$. The technology satisfies the following properties.

Assumption 2. *For all $j \in \mathcal{J}$,*

[3]This assumption is not as restrictive as it appears at first glance. If the technology is of constant returns to scale and inputs can be freely adjusted, the marginal contribution of a worker will be independent of the other inputs. The reason is that a profit-maximizing firm will keep the ratio between inputs constant.

i) $\sigma^j(a) > \sigma^j(h) > \sigma^j(g) \geq 0$ *and* $\sigma^j(a) > r$.

ii) $0 = \phi^j(a) < \phi^j(h) < \phi^j(g)$.

iii) $t_g^j > t_h^j$.

This implies that automation has a comparative advantage relative to human skills when skills are low, human skills have a comparative advantage when the skill is neither high nor low, and those in conjunction with generative AI have a comparative advantage when skills are high. Part iii says that output is maximized by automation when $t \leq t_h^j$, by a human-skills-only job when $t \in (t_h^j, t_g^j]$, and by generative AI-human skills job when $t > t_g^j$.

Rosen (1987) was the first to highlight the importance of non-pecuniary job characteristics in the compensating wage differentials literature. Lamadon, Mogstad, and Setzler (2022) show that worker preferences over non-pecuniary job characteristics lead to imperfect competition in the US labor market. Maestas, Mullen, Powell, von Wachter, and Wenger (2018) find that high-wage workers and college-educated workers have uniformly better job characteristics, and Mas and Pallais (2017) argue that there is evidence that workers in the US are willing to give up part of their income compensation to avoid undesirable working conditions. Sullivan and To (2014) show that there are substantial gains to workers from job search based on non-pecuniary factors, workers sort into jobs with better non-pecuniary job characteristics and are willing to pay for them. Sorkin (2018) shows a high prevalence of US workers that move to lower-paying firms in a way that cannot be accounted for by layoffs or differences in recruiting intensity to benefit from non-pecuniary job characteristics. He estimates that compensating differentials account for over half of the firm component of the earnings variance. These results, provide foundations for labor market power driven by the horizontal differentiation of jobs.

In addition to this, it is highly plausible that individuals with identical productivity may choose different jobs according to their differing tastes. Accounting for job preferences is particularly important to understand differences in job choices between different groups. For example, men and women exhibit different job choice patterns as well as Blacks and Whites.

3 The Equilibrium

3.1 Job Assignments

Because when a worker applies to a job in firm j, both the firm and the worker already know the worker's skill t, the firm chooses the job to maximize profits for each t; that is

$$\max_{\tau \in \{h,g\}} \{y^j(t;\tau) - w^j - r\mathbb{C}(\tau))\}. \tag{1}$$

where $\mathbb{C}(\tau) = 1$ if $\tau = g$ and zero otherwise.

Let $t^j(h,g)$ be the lowest skill level such that $y^j(t;g) - w^j - r \geq y^j(t;h) - w^j$. This leads to the following result.

Proposition 1. *The worker is assigned to job h if and only if $t \leq t^j(h,g)$ and to job g otherwise.*

Firm j places a worker with skill t in a job with generative AI when the difference between the net –of technological costs– productivity of human skills with generative AI and that without it is positive. Because the wage is already set, it does not affect the job allocation decision.

3.2 Equilibrium Wages

Let $d^j \in \{0,1\}$ be firm j's automation decision, where $d^j = 1$ means the job is assigned to human skills and $d^j = 0$ means the job is automated. Then, for any automation profile d, let $\mathcal{J}(d) \subseteq \mathcal{J}$ be the set of firms that opens a vacant and $\mathcal{J}(d^{-j}) \equiv \{k \in \mathcal{J} : k \neq j \text{ and } d^k = 1\}$ be the set of firm j's competitors.

Because workers observe (ϵ, w, t) before choosing a firm to supply their labor, they will choose the firm that provides the highest expected utility among all those that have a vacant available $j \in \mathcal{J}(d)$ provide that this yields a higher utility than the outside option. Thus, a worker chooses firm $j \in \mathcal{J}(t)$ whenever $w^j + \epsilon^j \geq \max\{b, w^{j'} + \epsilon^{j'}\}$.[4] Hence, the probability that a worker chooses firm $j \in \mathcal{J}(d)$ instead of any other firm is given by

$$P^j(w) = P[w^j + \epsilon^j \geq \max_{k \in \mathcal{J}(d^{-j})} \{w^k + \epsilon^k, 0\}] = \int_{\max\{\underline{\epsilon}, b - w^j\}}^{\bar{\epsilon}} \prod_{k \in \mathcal{J}(d^{-j})} G^k\Big(w^j + \epsilon^j - w^k\Big) dG^j(\epsilon^j),$$

[4]When there is no risk of confusion, we will omit the arguments to keep the notation simpler and we will omit the dependence of distributions and wages on the skill class s.

where the equality follows from the independence assumption about the Gs distributions.

Proposition 2. *$P^j(w)$ is strictly positive, strictly increasing in w^j, strictly decreasing in $w^{j'}$ for all $j' \neq j$, log-concave in w^j, and log-supermodular in w.*

For any given wage profile w, firm j's profits are then given by:

$$\Pi^j(w) \equiv \left(\int_{t^j(h,g)}^{\bar{t}} (y^j(t;g) - w^j - r) dF(t) + \int_{\underline{t}}^{t^j(h,g)} (y^j(t;h) - w^j) dF(t) \right) P^j(w). \quad (2)$$

Integrating by parts and simplifying, this can be written as follows

$$\Pi^j(w) \equiv \left(y^j(h,g) - w^j\right) P^j(w), \quad (3)$$

where

$$y^j(h,g) \equiv y^j(\bar{t};g) - \int_{t^j(h,g)}^{\bar{t}} \phi^j(g) F(t) dt - \int_{\underline{t}}^{t^j(h,g)} \phi^j(h) F(t) dt$$

is the net expected marginal product of labor.

Thus, firm j chooses w^j, taken $w^{-j} \equiv (\ldots, w^{j-1}, w^{j+1}, \ldots)$ as given, to solve the following problem

$$\max_{w^j \in \Re_+} \Pi^j(w^j, w^{-j}).$$

In what follows, we will focus on parametric restrictions such that the case in which $\underline{\epsilon} \leq b - w^j$ holds for all j and, therefore, the outside-option payoff is chosen with positive probability for each possible type.[5] From here onwards, let the subindex denote the derivative for the corresponding wage. Because G^k's are identically distributed, the first-order condition is given by

$$(y^j(h,g) - w^j) P^j_j(w) - P^j(w) \leq 0, \quad (4)$$

[5] This assumption does not change the results. If we allow for $\underline{\epsilon} > b - w^j$ in some occupations, then the mark-down will be a constant depending only on the number of firms.

where,

$$P_j^j(w) = \int_{b-w^j}^{\bar{\epsilon}} \sum_{h\in\mathcal{J}(d^{-j})} \nu_g\Big(w^j+\epsilon^j-w^h\Big) \prod_{k\in\mathcal{J}(d^{-j})} G\Big(w^j+\epsilon^j-w^k\Big)dG(\epsilon^j)+ \quad (5)$$
$$g(b-w^j)\prod_{k\in\mathcal{J}(d^{-j})} G(b-w^k),$$

where $\nu_g(\cdot) \equiv g(\cdot)/G(\cdot)$ is the of distribution G and the sub-index j denotes the derivative with respect to wage w^j.

Lemma 1. *Firm j's best response $B^j(w^{-j}) \in (0, y^j(h,g))$ exists and is unique.*

Profits are log-supermodular in w because the markdown depends only on w^j and $P^j(w)$ is log-supermodular in w. The following result readily follows from this and Theorem 6 in Milgrom and Roberts (1990). It also follows from Theorem 5 in Milgrom and Roberts (1990) that each firm has only one serially undominated strategy. Hence, the original game is dominance solvable and the equilibrium is globally stable under any adaptive learning rule satisfying assumption A6 in Milgrom and Roberts (1990).

Proposition 3. *For each s-type worker, the equilibrium set has the componentwise largest and smallest elements, given by $w_H(y,b)$ and $w_L(y,b)$ respectively, with*

$$w_l^j(y,b) = y^j(h,g)\frac{\xi^j(w_l(y,b))}{1+\xi^j(w_l(y,b))},$$

for all $j\in\mathcal{J}(d)$, where $\xi^j(w_l(y,b))$ is the elasticity of the labor supply for $l\in\{H,L\}$.

Hence, a type-s worker is paid a lower wage than his expected marginal product of labor. The mark-down as a percentage of the wage is the inverse of the labor-supply elasticity. The higher the elasticity; i.e., the more intense the competition, the higher the wage.

From here onwards, we will focus on the symmetric equilibrium for each type, which requires to assume that $y^j(h,g) = y(h,g),\ \forall\, j\in\mathcal{J}(s)$. Then, it readily follows from the first-order condition in equation (4) and integration-by-parts that the equilibrium wage $w(y,b)$ for an individual of type s is determined by a fixed point of the following equation

$$y(h,g) - w = m(b-w) \equiv \frac{1}{n}\frac{1-G(b-w)^n}{\underbrace{G(b-w)^{n-1}g(b-w)}_{\text{exclusion effect}} + \underbrace{\int_{b-w}^{\bar{\epsilon}} g(\epsilon)dG(\epsilon)^{n-1}}_{\text{competition effect}}}. \quad (6)$$

The numerator in equation (6) is the equilibrium labor supply since the workers choose the outside option with probability $G(b-w)^n$ (i.e., when each firm j has a valuation less than $b-w$). The denominator is the slope of the labor supply. This has two terms: (i) the market exclusion effect (equivalent to the exclusion effect in the goods market). When the valuations for all other firms are below $b-w$, which occurs with probability $G(b-w)^{n-1}$, firm j acts as a monopsony. Lowering its wage w by ϵ will exclude $\epsilon g(b-w)$ individuals from paid employment; and (ii) the competition effect (up to the adjustment that the marginal individual's valuation for paid employment is given by $b-w$) considering that a wage increase lowers the probability to be hired, which entails loosing not only the pecuniary benefit of being employed (w) but also the non-pecuniary benefit ϵ. So, this term is the density of a firm's marginal workers –those indifferent between the corresponding firm and the best outside option for them times the loss from a lower probability of being hired.

If both sides of equation (6) are divided by $w(y,b)$, the left-hand side is the Learner's index, denoted by $L(y) \equiv (y-w(y,b))/w(y,b)$, and the right-hand side is the inverse of the labor-supply elasticity, denoted by $\xi(y)$. Hence, in equilibrium, Lerner's index is the inverse of the supply elasticity. The Lerner's index ranges from 0 to ∞. A perfectly competitive firm pays $w(y,b)=y$, and therefore $L(y)=0$ –such a firm has no market power. An oligopsonist firm pays $w(y,b)<y$, so its index is $L(y)>0$, but the extent of its markdown depends on the elasticity of labor supply, which in turn depends on the strategic interaction with competing firms as well as the outside option.

Let $w^m(y,b)$ be the wage when there is a monopsony ($n=1$). In this case, the elasticity is equal to the hazard rate evaluated at $b-w^m(y,b)$ and this increases with $b-w^m$ due to the log-concavity of f. The following is proven in the appendix, where all proofs are placed.

Proposition 4. *For each s-type, there exists a unique symmetric equilibrium wage given by $w(y,b) \in [w^m(y,b),y]$.*

Uniqueness follows from the fact that f is log-concave which makes $r(b-w(y,b))$ increasing in $w(y,b)$, while the LHS in equation (6) is decreasing in $w(y,b)$. Log-concavity implies that the CDF of the second-order highest statistic increases in the sense of first-order stochastic dominance with $b-w(y,b)$. This explains why the RHS in equation (6) increases with $w(y,b)$ and is bounded. The LHS in equation (6) falls with w. Then by the Intermediate Value Theorem, there is a unique $w(y,b) \in [w^m(y,b),y]$ that solves equation (6).

Proposition 5. *The equilibrium wage $w(y,b)$ increases with (n,y,b) and $\lim_{n\to\infty} w(y,b) \to y$ and $\lim_{n\to\infty} P(w(y,b)) \to 0$.*

The equilibrium wage increases with competition intensity since workers are more likely to find another paid job that they like more than the one offered by firm j. This induces firm j to set a higher wage to attract workers. In the limit, when the number of firms goes to infinity, the worker is paid his marginal product of labor. This is due to log-concave distributions having either a fat or a thin tale. Otherwise, the wage markdown will not converge to zero as the number of firms approaches infinity (see, Gabaix, Laibson, Li, Li, Resnick, and de Vries (2016) for details).

Because an increase in competition intensity, holding wages constant, lowers the market exclusion effect since it increases the number of jobs available, and raises wages, employment at each firm decreases with competition intensity and Lerner's index falls.

An increase in the outside-option payoff raises the wage. The wage increases because workers choose the outside option more often if firms keep wages constant. Thus, firms increase wages less than the increase in b. The pass-trough from b to wages is equal to $-m'/(1-m')$, which is lower than 1.[6] Thus, a larger outside-option payoff decreases market power because, holding wages constant, the labor-supply elasticity raises as more workers find the outside option more attractive.

Corollary 1. *$w(y,b)$ increases with $(-r, \{(\sigma(\tau), \phi(\tau))\}_{\tau=\{h,g\}})$ and is non-decreasing with a first-order stochastic improvement in the skill distribution $F(t)$.*

3.3 Automation Decision

Firm j chooses to automate the job whenever this is more profitable than allocating the job to the worker. We will allow for mixed strategies $\alpha^j \in [0,1]$, where this is the probability of $d^j = 1$; i.e., to open a vacant.

Let firm j's expected profits from opening a vacant when competitors choose the mixed strategy α^{-j} be $\mathbb{E}_{\alpha^{-j}} \Pi^j(d^{-j})$, where $\mathbb{E}_{\alpha^{-j}}$ is the expectation with respect to d^{-j} under the mixed-strategy

[6] If firms could choose non-pecuniary benefits together with wages, they will also use them to compete against self-employment opportunities up to the point where the marginal return of increasing non-pecuniary benefits is equal to that from raising the wage.

profile α^{-j} and

$$\Pi^j(d^{-j}) \equiv \big(y^j(h,g) - w^j(y,b)\big) \int_{b-w^j(y,b)}^{\bar{\epsilon}} \prod_{k \in \mathcal{J}(d^{-j})} G\Big(w^j(y,b) + \epsilon^j - w^k(y,b)\Big) dG(\epsilon^j).$$

Firm j's best response is given by

$$BR^j(d^{-j}) \equiv \underset{\alpha^j \in [0,1]}{\operatorname{argmax}} \{\alpha^j \mathbb{E}_{\alpha^{-j}} \Pi^j(d^{-j}) + (1-\alpha^j)(\sigma(a) - r)\}.$$

This can be written as follows

$$BR^j(\alpha^{-j}) = \begin{cases} 1 & \text{if } \sigma^j(a) - r \leq \mathbb{E}_{\alpha^{-j}} \Pi^j(d^{-j}), \\ [0,1] & \text{if } \sigma^j(a) - r = \mathbb{E}_{\alpha^{-j}} \Pi^j(d^{-j}), \\ 0 & \text{if } \sigma^j(a) - r \leq \mathbb{E}_{\alpha^{-j}} \Pi^j(d^{-j}). \end{cases} \tag{7}$$

The next result readily follows from this and the Nash-equilibrium existence theorem.

Proposition 6. *For each worker's class, there exists a sub-game perfect equilibrium, denoted by* $(\alpha(y,b,r), w(y,b,r))$.

Let's assume symmetric firms and a symmetric equilibrium and consider $d^{-j} = \mathbf{1}$. Using the first-order conditions for wages, we deduce that firm j chooses $d^j = 1$ whenever

$$\sigma(a) \leq \sigma^o(a) \equiv \frac{1}{n^2} \frac{(1 - G(b-w)^n)^2}{G(b-w)^{n-1} g(b-w) + \int_{b-w}^{\bar{\epsilon}} g(\epsilon) dG(\epsilon)^{n-1}} + r.$$

Thus, we have the following result.

Proposition 7. *Let's consider a symmetric equilibrium.*

i) There exists a threshold $\sigma^o(a)$ such that for all $\sigma(a) > \sigma^o(a)$, the job is automatized.

ii) $\sigma^o(a)$ rises with (y,r) and falls with (n,b).

The second part is due to the pass-through from y and b to wages being positive and lower than 1, the equilibrium wage rises with n, and the labor supply, holding the wage constant, and the labor-supply elasticity falls with n.

3.4 Inefficient Job Assignments and Displaced Workers

In this section, we compare the job assignments already derived with the productive efficient job assignments when firms do not know the workers' productivity when choosing between automation and human-skills jobs. This counterfactual does not consider non-pecuniary benefits as the welfare-maximizing job assignment does.

The job assignment that maximizes expected output when the job is not automatized is a human-skill-only job whenever $t \leq t^j(h,g)$ and human-skills-generative-AI job otherwise. Because at the time the automation decision is made, firms do not know the workers' realized skills but know the productivity of automation, it is optimal to automatize the job whenever $\sigma^j(a) - r > y^j(h,g)$. We record this result in the next Lemma.

Lemma 2. *Firm j achieves the productive efficient job assignment if the job is automated whenever $\sigma^j(a) > y^j(h,g)+r$ and assigns the job to human-skill-only jobs whenever $\sigma^j(a) \leq y^j(h,g)+r$ and $t \leq t^j(h,g)$ and to human-skills-generative-AI jobs whenever $t > t^j(h,g)$.*

Let $\sigma^*(a) \equiv y^j(h,g) + r$. We deduce the next result from this and the ones in Lemma 1 and Proposition 7.

Proposition 8. *Suppose a symmetric equilibrium. If $\sigma(a) > \sigma^*(a)$, the job is efficiently automatized; if $\sigma^o(a) < \sigma(a) \leq \sigma^*(a)$, the job is inefficiently automatized, and if $\sigma(a) \leq \sigma^o(a)$, the job is efficiently assigned to human skills.*

The driving force behind this productive inefficiency is that firms choose automation versus human-skill jobs based on the rents they get from each option and not on productive efficiency. Thus, there is too much automation from the productive efficiency perspective. Because the wage is fixed when firms choose between a human-skill-only job and human-skill-generative-AI job, the job assignment when a worker is hired maximizes labor productivity; that is, workers with a comparative advantage in jobs complemented with generative AI are allocated to them while those with a comparative advantage in human-skill-only jobs are assigned to them.

The larger the rent the firm gets from human-skill jobs, the lower the productive inefficiency. This leads to the following result

Corollary 2. *The more competitive the labor market, the larger the productive inefficiency*

Competition between firms leads to an excessively high rate of job automation. Reallocating some workers to human-skill jobs would raise output by a quantity equal to the difference between the marginal product of labor in human-skills-only jobs and the productivity of automation. Displaced workers lose both the pecuniary benefits (wages) and the non-pecuniary benefits they would have gotten if those jobs had not been automated. The automation of these jobs creates an inefficiency because the rents firms would have gotten from hiring a worker are lower than the rents earned from automation but productivity would have been higher. The lower the rent from human skills; i.e., the more competitive the market, the higher the risk of being automated.

There are three reasons why productive inefficiency entails a welfare loss for the workers: firstly, they are paid less than the marginal product of labor; secondly, some workers lose the non-pecuniary benefits since they are inefficiently displaced by automation, and thirdly, workers are displaced and end up working in jobs where they are even less productive such as home production, self-employment, or unemployed.

3.5 Empirical Evidence

Acemoglu and Restrepo (2024), using data for the US from 1980 to 2016, estimates the impact of automation accounting for workers' rent dissipation on aggregates. This exercise suggests that the baseline ("competitive") effects of automation account for 42% of the increase in between-group inequality in the United States since 1980, while rent dissipation adds another 10 percentage points to automation's explanatory power for between-group inequality and is responsible for pushing several demographic groups from stagnant into negative real wage changes. They also estimate that because of worsening allocative efficiency, automation brought small gains in TFP, average wages, and consumption since 1980.

Aghion et al. (2022) find, using French manufacturing firms, that the employment response to automation remains positive at the industry level despite the potential for business stealing effect. They provide evidence consistent with the view that the business-stealing effect induced by automation mainly affects foreign competitors' employment in sectors facing international competition, whereas it mainly affects domestic competitors' employment in less open sectors. At the firm-level, there is no such heterogeneity and the response of employment and sales remains positive and significant regardless of the degree of exposure to external competition.

4 Taxing Technological Capital

Keynes (1929) predicted that the rapid spread of technologies would bring "technological unemployment". Leontief made a similar prediction: "Labor will become less and less important... . More and more workers will be replaced by machines. I do not see that new industries can employ everybody who wants a job".

These ideas were echoed by business people and politicians, who argue about the potential benefits of taxing automation based on the belief that it will lead to large job losses and lower wages.

To study this claim let's consider a benevolent social planner who chooses automation to maximize profits plus workers' surplus; that is $\max_{d\in\{0,1\}^n}\{W(d)\}$, where

$$W(d) = \max\Big\{\sum_j \Pi^j(d^{-j}) + V(d), \sum_j (y^j(0,a)r) + b\Big\}$$

and

$$V(d) \equiv \sum_{j\in\mathcal{J}(d^{-j})} \int_{b-w^j}^{\bar{\epsilon}} (w^j + \epsilon^j) \prod_{k\in\mathcal{J}(d^{-j})} G\big(w^j + \epsilon^j - w^k\big) dG(\epsilon^j) \tag{8}$$

To simplify the analysis, let's focus on a symmetric equilibrium. Thus, welfare is given by

$$W(d) = \max\left\{n\int_{b-w}^{\bar{\epsilon}} (y(h,g)+\epsilon)G^{n-1}(\epsilon)g(\epsilon)d\epsilon, n(y(0,a)-r)+b\right\}.$$

After integration-by-parts, the first term in the maximum in the welfare function is given by[7]

$$y(h,g) + \bar{\epsilon} - (y(h,g)+b-w)G^n(b-w) - \int_{b-w}^{\bar{\epsilon}} G(\epsilon)^n d\epsilon.$$

It readily follows that efficiency requires paying workers their marginal product of labor since welfare raises with w. This leads to the following result

[7]Observe that integration-by-parts implies the following

$$n\int_{b-w}^{\bar{\epsilon}} \epsilon G(\epsilon)^{n-1} g(\epsilon)d\epsilon = \bar{\epsilon} - (b-w)G^n(b-w) - \int_{b-w}^{\bar{\epsilon}} G(\epsilon)^n d\epsilon.$$

Proposition 9. *Suppose firms are symmetric. Then, there exists a threshold $\sigma^{**}(a)$, given by*

$$-\frac{b}{n}G^n(b-y(h,g))+\frac{1}{n}\int_{b-y(h,g)}^{\bar{\epsilon}}(1-G(\epsilon)^n)d\epsilon+r.$$

such that automation is efficient whenever $\sigma(a)>\sigma^{**}(a)$. In addition, $\sigma^{**}(a)>\sigma^{o}(a)$

The second part follows from the fact that welfare is the sum of profits plus workers' expected utility and this exceeds b. Thus, the rent from automation must be larger for automation to be welfare maximizing than for it to achieve productive efficiency. This happens because when a job is automated, its productivity and non-pecuniary benefits are lost and the latter are considered only under welfare maximization.

Because there is an inefficiently high level of automation, levying different percentual tax/subsidy of $\eta \in \Re$ per-dollar on technological capital may help to restore the productive and allocative inefficiency.

Let's define $\Pi(\eta)$ as the symmetric-equilibrium profits when the tax rate is η ad $\bar{\eta}$ as the largest η such that $\Pi(\eta)=0$. Also, let $W(1)$ be the total welfare when the job is allocated to human skills.

Proposition 10. *Suppose firms are symmetric. If $\bar{\eta}>\frac{1}{r}W(1)$, for any $\sigma(a)\in[\sigma^{o}(a),\sigma^{**}(a)]$, there exists a tax rate η^{**} such that $\sigma(a,\delta)=\sigma^{**}(a)$ and if $\bar{\eta}>\frac{1}{r}y(h,g)$, for any $\sigma(a)\in[\sigma^{o}(a),\sigma^{*}(a)]$, there exists a tax rate η^{*} such that $\sigma(a,\delta)=\sigma^{*}(a)$. However, the tax reduces labor productivity since the adoption of generative AI is inefficiently low.*

Because a human-skill-generative-AI job is chosen when it maximizes productivity, a tax on technological capital results in a lower marginal product of labor since adopting generative AI becomes onerous. The tax also lowers the equilibrium wage and therefore the outside option is taken more often with the corresponding extra loss in expected non-pecuniary benefits.

5 Automation and Skill Acquisition

In this section, we study how labor market power and automation influence workers' incentives to invest in skills. We will assume that workers can invest in human capital (skills) before firms make automation decisions. Namely, Workers choose s at a cost $c(s)$. This is increasing, convex, and $c(0)=0$.

Workers choose s to maximize expected utility; that is, solve $\max_{s\in\Re_+}\{\max\{b, U(w(y,b))\} - c(s)\}$. For a non-displaced worker, $\max\{b, U(w(y,b))\} = U(w(y,b))$ since he can always choose to take b after wages are set. A displaced worker is forced to take his outside-option payoff.

Because $y(h,g)$ rises with an increase in s since this implies a FOSD improvement in $F(t|s)$, and the pass-through from $y(h,g)$ to wages is lower than 1, the rent from human-skill jobs increases with s. This implies that $\sigma^o(a)$ falls with s.

Let's define $s(a)$ as the lowest skill level such that $\sigma^o(a) \geq \sigma(a)$. Because $\max\{b, U(w(y,b))\} = U(w(y,b))$ for all $s \geq s(a)$ and $\max\{b, U(w(y,b))\} = b$ otherwise, the first-order condition is as follows

$$\begin{cases} \frac{y_s(h,g)}{1-m'(b-w(y,b))}(1 - G^n(b - w(y,b))) - c_s(s) \leq 0 & \text{if } s \geq s(a), \\ 0 - c_s(s) \leq 0 & \text{if } s < s(a). \end{cases} \tag{9}$$

The first-order condition is explained by the fact that the worker chooses paid employment with probability $1 - G^n$ and the pass-through from y to wages is $1/(1 - m') < 1$. Hence, he does not fully internalize the whole return on his investment. Because of this, the worker's incentives, ceteris paribus, to improve his skills are low relative to his incentives in a competitive market since he is the full residual claimant on the return to the investment when the market is competitive.

Let's denote the solution to the first-order condition when $s \geq s(a)$ by $s(h,g)$. Then if $s(h,g) \geq s(a)$, assuming quasi-concavity of $U(w)$ in s, $s(h,g)$ is the optimal investment. Otherwise, a worker prefers not to invest in skills and to be displaced whenever the payoff from that is higher than the payoff from investing the minimum required to stop the firm from automating the job; that is, to invest up to the point where $s = s(a)$. This happens when the following holds

$$b \geq \Big(w + G(b-w)^n(b-w) + n\int_{b-w}^{\bar{\epsilon}} \epsilon g(\epsilon)G(\epsilon)^{n-1}d\epsilon - c(s)\Big)\Big|_{s=s(a)}. \tag{10}$$

Otherwise, the worker prefers to invest up to the point where $s = s(a)$.

When $s(a) > s(h,g)$, $U(w(y,b)) - c(s)$ falls with s and thereby there is a skill level $s(b) > s(h,g)$ such that $U(w(y,b)) - c(s) \leq b$ for all $s \geq s(b)$. Thus, if $s(b) > s(a)$, a worker will prefer to invest up to $s = s(a)$ and to avoid being displaced than to invest zero, to be displaced, and to get a payoff equal to b.

Because $s(a)$ increases with b, we deduce the following result.

Proposition 11. *Suppose that $U(w(y,b)) - c(s)$ is quasi-concave in s. Then, if $s(a) \leq s(h,g)$, the optimal investment in skills is $s^o = s(h,g)$. Otherwise, there exists a threshold $\tilde{b}$ such that$s^o = s(a)$*

if and only if $b \leq \tilde{b}$ *and* $s^o = 0$ *otherwise.*

When the skill level required to prevent firms from automating jobs ($s(a)$) is lower than the skill level that maximizes the expected utility when the job is not automated ($s(h,g)$), workers invest $s(h,g)$. If not, they will invest up to $s(a)$ when the outside-option payoff b is smaller than the expected utility evaluated at $s = s(a)$. In contrast, if b exceeds the expected utility evaluated at $s = s(a)$, workers will not invest in skills, as they prefer being displaced rather than bearing the cost of additional skill acquisition.

Because workers are not full residual claimants on the return to skills, they underinvest relative to the level consistent with productive efficiency whenever $s(h,g) \geq s(a)$. However, when $s(h,g) < s(a) \leq s(b)$, workers' investment could prevent automation and this could mitigate workers' incentive to underivest and may even create incentives to overinvest relative to the efficient level. This leads to the following result.

Proposition 12. *If either* $s(a) \leq s(h,g)$ *or* $b > \tilde{b}$*, there is underinvestment in skills, while if* $s(a) > s(h,g)$ *and* $b \leq \tilde{b}$*, there is overinvestment in skills.*

Workers with a low outside-option payoff are more likely to overinvest in skills relative to workers with a higher outside-option payoff because they benefit relatively more from preventing automation.

The evidence points towards an increase in human capital investment for those who are more exposed to automation. However, by increasing training they might not stop firms from automating jobs but rather induce more hiring in other jobs where the acquired skills are productive. For instance, HeB, Janssen, and Leber (2023) find that workers exposed to substitution by automation are 15 percentage points less likely to participate in training than those not exposed to it. In addition, workers who leave occupations highly exposed to automation increase their training participation, while those who enter them train consistently less. The automation training gap is particularly pronounced for medium-skilled and male workers, and is largely driven by the lack of ICT training and training for soft skills. Moreover, workers in exposed occupations receive less financial and non-financial training support from their firms, and the training gap is almost entirely related to a gap in firm-financed training courses.

Dauth, Findeisen, Suedekum, and Woessner (2021) find that robots' adoption is associated with displacement effects in manufacturing but these are fully offset by new jobs in services. The most affected are young workers just entering the labor force. Automation is related to more stable employment within firms for incumbents, and this is driven by workers taking over new tasks in their original plants. However, young workers change their human capital investment strategy away from vocational training and towards colleges and universities.

Innocenti and Golin (2022) find, using data from representative samples of working individuals in 16 countries, that workers' intentions to invest in training outside their workplace–controlling for other behavioral traits– raise with the fear of automation. They also report that fear of automation reinforces the effect that internal locus of control exerts on retraining intentions.

6 Conclusions

This paper argues that when labor markets are non-competitive, firms automate more jobs than productive and welfare efficiency require. Conditional on automation not taking place, the adoption of generative AI accords with productive efficiency; that is, when generative AI has a comparative advantage relative to solo human-skill jobs. This happens because firms with market power choose automation by comparing the rent from hiring a worker to the rent from automating the job.

Because automation is adopted more frequently than efficiency demands, we argue that a tax on technological capital can solve the productive and welfare inefficiency due to automation. However, it gives rise to productive and welfare inefficiency in the adoption of generative AI. Thus, taxing technological capital must be done by trading off the inefficiency due to excessive automation against the inefficiency due to the induced shortage of jobs where generative AI is adopted and the concurrent wage loss.

Last but not least, we argue that workers who anticipate being displaced may overinvest in human capital to increase the rent from human-skill jobs, making automation relatively less profitable. However, workers from low-skill classes might be completely discouraged from investing in skills since they anticipate their jobs will be automated even when they invest in human skills.

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A Appendix

Proof of Lemma 1. Existence follows from Weiertress' Theorem and uniqueness from the log-concavity of the profit functions: the best response is the solution to

$$P_j^j(w) - \frac{1}{y^j(h,g) - w^j} = 0. \tag{A1}$$

It follows from this that profits are log-concave whenever

$$\frac{P^j(w)P_{jj}^j(w) - (P_j^j(w))^2}{(P^j(w))^2} - \frac{1}{(y^j(h,g) - w^j)^2} \leq 0, \tag{A2}$$

where the inequality follows from the fact that $P^j(w)$ is log-concave in w □

Proof of Proposition 4. The proof of this result follows closely Zhou (2017).

Recall that the first-order condition is given by

$$y(h,g) - w = \frac{1}{n}\frac{1 - G(b-w)^n}{G(b-w)^{n-1}g(b-w) + \int_{b-w}^{\bar{\epsilon}} g(\epsilon)dG(\epsilon)^{n-1}}. \tag{A3}$$

Lets define CDF of the second-highest order statistics by

$$G_{n-1}(b-w) = G(b-w)^n + nG(b-w)^{n-1}(1 - G(b-w))$$

Observe that at $w = y$, the right-hand side equation (A3) is greater than the left-hand side since it is strictly positive. Let $\lambda_g(\cdot) \equiv g(\cdot)/(1 - G(\cdot))$ be the hazard rate. Let's define $w^m(y)$ as the wage when there is only one firm. It is easy to check that this is the unique (due to log-concavity of $g(\cdot)$) solution to the following equation

$$y - w^m = \frac{1}{\lambda_g(y - w^m)}.$$

Observe that at $w = w^m$, the LHS is larger than the RHS. To see this notice that

$$\begin{aligned} m(b-w) =& \frac{1 - G(b-w)^n}{nG(b-w)^{n-1}g(b-w) + \int_{b-w}^{\bar{\epsilon}} \lambda_g(\epsilon) dG_{n-1}(\epsilon)} \\ <& \frac{1 - G(b-w)^n}{nG(b-w)^{n-1}g(b-w) + \lambda_g(b-w)(1 - G_{n-1}(b-w))} \\ =& \frac{1}{\lambda_g(b-w)}, \end{aligned}$$

where the inequality follows from the fact λ_g is increasing. One deduces from this and the definition of $w(y, b)$, that the left-hand side is greater than the right-hand side at $w(y, b) = w^m(y, b)$. Thus, the first-order condition in equation (A3) has a solution $w \in [w^m(y, b), y]$.

Because the left-hand side of the first-order condition in equation (A3) falls with w at rate equal to -1, the solution is unique if $m'(y - w) < 0$. To show that this is the case observe that

$$m'(b-w) = -n \frac{G(b-w)^{n-1} g(b-w)}{1 - G(b-w)^n} m(b-w) \left(1 + m(b-w) \frac{g'(b-w)}{g(b-w)}\right).$$

Thus, $m'(b-w) < 0$ if and only if

$$ng(b-w)\Big(G(b-w)^{n-1}g(b-w) + \int_{b-w}^{\bar{\epsilon}} g(\epsilon) dG(\epsilon)^{n-1}\Big) + (1 - G(b-w)^n) g'(b-w) > 0.$$

Using the fact that log-concavity implies that $(1-G)g' + g^2 > 0$, one deduces that the inequality above holds if

$$n \int_{-w}^{\bar{\epsilon}} g(\epsilon) dG(\epsilon)^{n-1} > (1 - G(b-w)^n) \lambda_g(b-w) - nG(b-w)^{n-1} g(b-w).$$

Using the definition of G_{n-1}, we re-write this as follows

$$\int_{-w}^{\bar{\epsilon}} \lambda_g(\epsilon) dG(\epsilon)_{n-1} > (1 - G_{n-1}(b-w)) \lambda_g(b-w).$$

The inequality holds because the hazard rate is increasing.

□

Proof of Proposition 2. Because G^k are identically distributed, for all $j \in \mathcal{J}(d)$

$$P^j(w) = \int_{b-w^j}^{\bar{\epsilon}} \prod_{k \in \mathcal{J}(d^{-j})} G^k\Big(w^j + \epsilon^j - w^k\Big) dG^j(\epsilon^j),$$

and for all for all $j \in \mathcal{J}(d)$,

$$P^j_j(w) = \int_{b-w^j}^{\bar{\epsilon}} \sum_{h \in \mathcal{J}(d^{-j})} \nu_g\Big(w^j + \epsilon^j - w^h\Big) \prod_{k \in \mathcal{J}(d^{-j})} G\Big(w^j + \epsilon^j - w^k\Big) dG(\epsilon^j) + \\ g(b - w^j) \prod_{k \in \mathcal{J}(d^{-j})} G(b - w^k),$$

where $\nu_g(\cdot) \equiv g(\cdot)/G(\cdot)$,

$$P^j_h(w) = -\int_{b-w^j}^{\bar{\epsilon}} \nu_g\Big(w^j + \epsilon^j - w^h\Big) \prod_{k \in \mathcal{J}(d^{-j})} G\Big(w^j + \epsilon^j - w^k\Big) dG(\epsilon^j) < 0.$$

$P^j(w)$ is strictly increasing in w^j and is strictly decreasing in $w^{j'}$.

$P^j(w)$ is log-concave in w^j if and only if the following holds

$$\frac{1}{P^j(w)} P^j_{j,j}(w) - \frac{1}{(P^j(w))^2} (P^j_j(w))^2 \leq 0$$

This holds because the multitplication of log-concave functions is log concave and $G()$ is log-concave in w^j.

Also observe that for all $j \in \{1, \ldots, n\}$, we have that for all $j \in \mathcal{J}(d)$

$$P^j_{jj}(w) = \int_{b-w_j}^{\bar{\epsilon}} \left(\sum_{h \in \mathcal{J}(d^{-j})} \nu'_g\Big(w_j + \epsilon_j - w_h\Big) + \Big[\sum_{h \in \mathcal{J}(d^{-j})} u_g\Big(w_j + \epsilon_j - w_h\Big) \Big]^2 \right) \times \\ \prod_{k \in \mathcal{J}(d^{-j})} G\Big(w_j + \epsilon_j - w_k\Big) dG(\epsilon_j) - g'(b - w_j) \prod_{k \in \mathcal{J}(d^{-j})} G(b - w_k).$$

Observe that for all $j, h \in \mathcal{J}(d)$, log-supermodularity implies

$$\frac{1}{P^j(w)} P^j_{j,j'}(w) - \frac{1}{(P^j(w))^2} P^j_j(w) P^j_{j'}(w) \geq 0.$$

Observe that

$$P^j_{jh}(w) = \int_{b-w_j}^{\bar{\epsilon}} \Bigg(-\nu_g'\Big(w_j+\epsilon_j-w_h\Big) - \nu_g\Big(w_j+\epsilon_j-w_h\Big) \sum_{h\in\mathcal{J}(d^{-j})} \nu_g\Big(w_j+\epsilon_j-w_h\Big)\Bigg)\times$$
$$\prod_{k\in\mathcal{J}(d^{-j})} G\Big(w_j+\epsilon_j-w_k\Big)dG(\epsilon_j) + \nu_g\big(b-w^h\big) \prod_{k\in\mathcal{J}(d^{-j})} G\big(-w^k\big)g\big(b-w^j\big)$$

since $\nu_g' < 0$ because $g(\cdot)$ is log-concave.

□

Proof of Proposition 5. It follows from the first-order condition in equation (6) and uniqueness that the equilibrium wage increases with $x \in (y, b)$ if and only if

$$\frac{\partial w}{\partial x}(1-m'(b-w)) = \frac{\partial y}{\partial x} - m'(b-w)\frac{\partial b}{\partial x} > 0. \tag{A4}$$

Because $m'(\cdot) < 0$, we deduce that $w(y,b)$ rises with (y,b).

Next, we show that $w(y,b)$ increases with n and converges to y as n goes to infinity. Observe that (6) rewrites as follows

$$\frac{1}{y-w} = \frac{g(\bar{\epsilon}) - \nu_g(b-w)G(b-w)^n - \int_{b-w}^{\bar{\epsilon}} g'(\epsilon)G(\epsilon)^{n-1}d\epsilon}{(1-G(b-w)^n)/n}$$
$$= n\frac{g(\bar{\epsilon}) - \nu_g(b-w)G(b-w)^n}{1-G(b-w)^n} - \int_{b-w}^{\bar{\epsilon}} \frac{g'(\epsilon)}{g(\epsilon)} d\frac{G(\epsilon)^n - G(b-w)^n}{1-G(b-w)^n},$$

where the first step follows from integration by parts. One can show that the first term rises with n. Second, log-concavity of $g(\cdot)$ implies that $-\frac{g'}{g}$ is increasing. Third, $\frac{G(\epsilon)^n - G(b-w)^n}{1-G(b-w)^n}$ is the distribution of the highest order statistics conditional on this being greater than $b-w$ and, therefore, it increases in n in the sense of first-order stochastic dominance. The result follows from these three facts.

Observe that

$$\lim_{n\to\infty} \frac{g(\bar{\epsilon}) - \nu_g(b-w)G(b-w)^n - \int_{b-w}^{\bar{\epsilon}} g'(\epsilon)G(\epsilon)^{n-1}d\epsilon}{(1-G(b-w)^n)/n} \to \infty$$

and therefore $\lim_{n\to\infty} w(y,b) \to y$. This follows from the fact that the numerator goes to $g(\bar{\epsilon})$, while the denominator goes to 0 due to the fact that $\lim_{n\to\infty} G(b-w)^n \to 0$.

Observe that

$$\frac{\partial w}{\partial n} = \frac{m(b-w)}{1-m'(b-w)}\left(\frac{1}{n} + \frac{\ln G(b-w)}{1-G(b-w)^n} + \right.$$
$$\left. n\frac{m(b-w)}{1-G(b-w)^n}\int_{b-w}^{\bar{\epsilon}} \nu_G(\epsilon)g(\epsilon)G(\epsilon)^{n-1}\big(1-(n-1)(\ln G(b-w) - \ln G(\epsilon))\big)d\epsilon\right) > 0$$

Observe that the sum of the first two terms can be re-written as follows: $\frac{1-G^n}{n} + G^n \log F$, When evaluated at $\bar{\epsilon}$, this is zero, while at $-\bar{\epsilon}$, it is equal to 1 since $G^n \log G$ goes to zero (by L'Hopital). The derivative of this with respect to ϵ is given by $n^2 G^{n-1} g \log G$, which is 0 at $-\bar{\epsilon}$ and strictly negative in $(-\bar{\epsilon}, \bar{\epsilon}]$ and therefore $\frac{1-G^n}{n} + G^n \log G \geq 0$. This together with the fact that $\ln G(b-w) - \ln G(\epsilon) \leq 0$ for all $\epsilon \geq b-w$ proves the result. ☐

Proof of Proposition 10.

$$\sigma(a) - (1+\eta)r \leq \frac{1}{n^2}\frac{(1-G(b-w(\eta))^n)^2}{G(b-w(\eta))^{n-1}g(b-w(\eta)) + \int_{b-w(\eta)}^{\bar{\epsilon}} g(\epsilon)dG(\epsilon)^{n-1}}. \tag{A5}$$

Let's define $\sigma(a,\eta)$ as the automation productivity that leaves the firm indifferent between automation and human-skill jobs. Thus, $\sigma(a,\eta) = (1+\eta)r + \Pi(\eta)$.

Observe that the RHS of inequality in (A5) falls with the tax rate whenever

$$\frac{\partial w}{\partial \eta}\Big(G(b-w(\eta))^{n-1}g(b-w(\eta))m(b-w) - \frac{1-G(b-w(\eta))^n}{n}m'(b-w)\Big) < 0.$$

This holds since $m' < 0$ and the wage falls with η.

Substituting for $m'()$ and the derivative of the wage, we can show the LHS of equation A5 falls with η at a faster rate than the RHS of it whenever

$$\frac{G(b-w(\eta))^{n-1}g(b-w(\eta))m(b-w)}{1-m'(b-w)}\Big(2 + \frac{g'(b-w(\eta))}{g(b-w(\eta))}m(b-w)\Big) < 1.$$

Because

$$1-m'(b-w) = \frac{1}{1-G(b-w)^n}\left(1-G(b-w)^n + nG(b-w)^{n-1}g(b-w)m(b-w)\left(1+m(b-w)\frac{g'(b-w)}{g(b-w)}\right)\right).$$

ànd $G(b-w)^{n-1}g(b-w)m(b-w) < 1$, the inequality holds.

Next, notice that $\sigma(a,\eta)$ rises continuously with η. The tax rate such $\sigma(a,\eta) = \sigma^{**}(a)$ is given by the solution of the following equation

$$\eta = \frac{1}{r}\big(\Pi(1) + V(1) - \Pi(\eta)\big),$$

where $\Pi(1) + V(1)$ is teh welfare from a human-skill job. Observe that at $\eta = 0$, the LHS is lower than the RHS. The LHS raises at rate 1 and the RHS increases at rate lower than 1 since RHS of inequality in (A5) falls with η at rate lower than r. Let's define $\bar{\eta}$ as the largest η such that $\Pi(\eta) = 0$. Then if $\bar{\eta} > \frac{1}{r}\big(\Pi(0) + V(0)\big)$, by the Intermediate Value theorem there is a tax rate such that $\sigma(a,\eta) = \sigma^{**}(a)$.

Next consider productive efficiency. Recall that $\sigma(a,\eta) = (1+\eta)r + \Pi(\eta)$ is the automation productivity that leaves the firm indifferent between automation and human skills. Productive efficiency requires $\sigma^{*}(a) \equiv y^{j}(h,g) + r$. Let's define η^{*} as the solution to $(1+\eta)r + \Pi(\eta) = y^{j}(h,g) + r$. If $\bar{\eta} > \frac{1}{r}y(h,g)$, following the same steps as above we can show the existence of η^{*}. □

Proof of Proposition 12. Observe that $\sigma^{o}(a)$ falls with b since the rent from human skills falls with b. This occurs because $b - w(y,b)$ raises with b due to the pass-trough from b to w is $-m'/(1-m') < 1$. Because $s(a)$ solves $\sigma^{o}(a) \geq \sigma(a)$ and $\sigma^{o}(a)$ falls with b, $s(a)$ increases with b.

Observe also that the LHS in equation (10) raises with b at rate 1, and RHS does so at a rate $w_b + G^{n}(1-w_b) + U_s(w)s_b(a)$, with $U_s(w)s_b(a) < 0$, which is lower than 1, we deduce that $s(b)$ increases with b.

Because $s(0) = 0$ and $s(a)|_{b=0} > 0$ and for b sufficiently large $s(b) > s(s)$, there exists a $\tilde{b}$ such that $s(b) > s(s)$ for all $b > \tilde{b}$.

□