Investments, Competition, and Endogenous Cash Constraints

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Abstract

This paper studies the impact of external financing on firms' incentives to invest in a general borrowing-then-investment oligopoly game. Firms choose to finance their investment by means of a standard debt contract. When the strategic-commitment effect of debt is non-negative, firms borrow sufficient funds to avoid being cash-constrained in the investment sub-game. Conversely, when the strategic-commitment effect of debt is negative, firms borrow an amount that leaves them cash-constrained during the investment sub-game. Consequently, being cash-constrained in the investment stage is a strategic choice. In this case, policies aimed at alleviating firms' access to external funds would not lead to increased investments.

Keywords: Financial Constraints, Debt Contracts, Competition, Industry-wide Investment, complementarities, Spillovers.

JEL-Classification: G3, L12, L13, L41, D24, O31

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1 Introduction

The evidence regarding the impact of financial structure and constraints on firms' investment decisions is extensive. For instance, Denis and Sibilkov (2009) find that cash holdings have a significantly greater effect on firm value for constrained firms compared to unconstrained firms and are more positively associated with net investment (capital expenditures net of depreciation) for firms with high hedging needs. Additionally, Kaplan and Zingales (1997) find that firms classified as less financially constrained actually exhibit greater investment-cash-flow sensitivity.[1]

Brown and Petersen (2011) emphasize the role of internal finance in driving innovation for small, high-tech firms due to capital market imperfections. Several papers have shown that credit constraints hinder innovation, utilizing various datasets, industries, measures of R&D, credit constraints, and methodologies to address endogeneity.[2]

Moreover, evidence suggests that firms' capital structure choices are influenced by the level of competition in the product market, which determines competition intensity during investment decisions and competitors' financial decisions. Survey evidence, such as that from Graham and Harvey (2001), indicates that CFOs consider competitors' financing decisions in shaping their own. Empirical studies, such as Valta (2012), demonstrate that industry average leverage ratios significantly impact firms' capital structures, with the cost of bank debt being higher for firms in competitive markets. Additionally, research by Boubaker, Saffar, and Sassi (2018) finds that competitive pressure from product markets leads firms to rely less on bank debt financing.[3]

Most existing research assumes, based on Modigliani and Miller (1958), that capital structure and investment choices are made independently across firms, despite evidence to the contrary. Specifically, factors such as the marginal tax rate, expected deadweight loss in default, information asymmetries, investment profitability, and the incentive structure typically influence capital structure and investment decisions.

The existing literature examines two avenues through which independence is compromised. Firstly, in more competitive markets, firms have incentives to take on debt and increase production to gain a strategic advantage over industry rivals. Secondly, a firm's asset liquidation value rises in more competitive markets, influencing both the number and financial strength of potential buyers for liquidated assets. However, as

[1]See, also, Almeida, Campello, and Weisbach (2011b) and Almeida, Campello, and Hackbarth (2011a).

[2]See, for instance, Brown and Petersen (2011), Brown, Martinsson, and Petersen (2012), Chava, Oettl, Subramanian, and Subramanian (2013), Gorodnichenko and Schnitzer (2013), Nanda and Nicholas (2014), Hsu, Tian, and Xu (2014), Cerqueiro, Hegde, Penas, and Seamans (2017), Mann (2018), and Giebel and Kraft (2019). Kerr and Nanda (2015) and Hall and Lerner (2010) provides extensive reviews of this literature.

[3]See, also, Chevalier (1995), Phillips (1995), Kovenock and Phillips (1997)). Campello (2006),Akdoğu and MacKay (2008), and Xede, Simon Peter Dak-Adzaklo, Ofosu, and Wise Dodzidenu Adza (2023).

most firms operate in oligopolistic markets, independence is inherently compromised. The strategic relationship between investment decisions becomes pivotal in determining industry-wide aggregate investment in an industry equilibrium. This strategic interaction is critical for managers in the context of capital budgeting, as competitors' investment responses impact the returns of the firm's own investment, influencing firms' financing decisions and financial structure.

This paper examines the relationship between market structure, capital structure, and investment decisions within a three-period oligopoly game involving two or more potentially heterogeneous firms. During the initial phase, firms simultaneously choose their optimal financial structure and determine the amount to borrow. Subsequently, firms simultaneously make decisions regarding the amount to invest, considering their cash constraints. In the final period, payoffs are realized, and repayments are made.

The paper yields three main insights: Firstly, it highlights the intricate interplay between firms' capital structure, investments, and product market decisions, even in the absence of agency conflicts, tax benefits, and bankruptcy costs. Secondly, it asserts that the standard debt contract, which satisfies limited liability and monotonicity, represents the optimal financial arrangement. Thirdly, it suggests that, under certain conditions related to investment externalities and complementarities, firms may strategically utilize debt to self-impose cash constraints during the investment period. Consequently, while the model's findings align with the empirical observation that being cash-constrained limits firms' investment capabilities, it questions the widely accepted belief that policies aimed at easing financial constraints are the sole solution to the underinvestment issue.

Each firm possesses a profit function and internal cash, with profits being verifiable and available for pledging to lenders. Therefore, firms realize returns on their investments only after fulfilling all repayment obligations. The profit functions result from an unmodeled product-market game occurring subsequent to the investment subgame and are contingent upon the investment profile and a profitability shock, such as a demand or cost shock. Profits increase with a firm's own investment and both the firm's profits and the marginal return to the firm's own investment rises with the profitability shock.

To accommodate various oligopoly games featuring homogeneous and differentiated products, as well as firm heterogeneity, the profit functions allow for any pair of investments to act as complements or substitutes. Moreover, investments can yield positive or negative externalities, reflecting the impact of a firm's investment on its competitors' profits. These externalities stem from technology and product market spillovers, with technology spillovers affecting marginal costs, demand, or both. The strategic-commitment effect of investments in the product-market game also influences the complementarity/substitutability of investments as well as the sign and magnitude of the externalities. For instance, in Cournot and Bertrand duopoly games with differentiated goods, investments are complements with positive externalities when cost spillovers exceed an increasing function of the product differentiation parameter; otherwise, they act as substitutes with negative externalities. In the Hotelling's game, investments are substitutes with negative externalities irrespective of the feasible spillover and competition intensity. The market structure remains exogenous and constant throughout the analysis.

In the investment sub-game, firms invest the minimum between their owned and borrowed cash and the amount that maximizes, net of repayment obligation, profits. Consequently, cash-constrained firms underinvest. Importantly, their investment remains unaffected by competitors' investment levels. Conversely, unconstrained firms' investment increases with competitors' investment when investments are complements, but it might decrease when they are substitutes. In the investment sub-game, cash does not play a strategic role since firms cannot commit to deviating from the investment strategy that maximizes profit.

Given two financial contracts with identical expected repayment that satisfy limited liability and monotonicity –where repayments are non-decreasing with realized profits– firms invest more and earn more with the contract that repays less in high-profitability states and more in low-profitability states. The rationale behind this result is that the former allows the firm to appropriate a larger share of the return on its investment in those states where this is more profitable. This implies that firms benefit more from a standard debt contract that pays the full return up to a certain profit level and then a fixed amount thereafter, compared to an equity financing contract that repays a constant share of profits in each state.

An exogenous positive cash shock to a cash-constrained firm, resulting from an increase in internal cash, external cash (external financing), or both, leads to an increase in its investment, regardless of whether investments are complements or substitutes. This is because the firm's initial situation entails underinvestment due to its cash constraint binding, which is independent of competitors' investments. Cash shocks do not alter competitors' best responses, so a positive cash shock increases investment for unconstrained competitors when investments are strategic complements, since best responses are non-decreasing. Hence, industry-wide investment rises. Nevertheless, when investments are substitutes, the impact of cash shocks on competitors' and industry-wide investment is ambiguous, as the best-response functions are decreasing. Within the framework of the implicit function theorem, it is possible to provide a sufficient condition for industry-wide investment to weakly increase with any firm's cash, regardless of whether some or all firms' investments are substitutes or complements.[4]

[4]The sufficient condition is based on Christensen (2018) who argue that the negative of the Jacobian of firms' first-order conditions must be a B_0-matrix. This condition, which is less restrictive than the standard dominant-diagonal condition and a proper subset of the set of B_0 matrices, assumes differentiability and applies only when an infinitesimal change in a firm's cash does not cause an unconstrained firm to become constrained or vice versa.

In the first stage, firms offer external financiers a contract that provides them with zero expected profits. When the ratio between the marginal profitability of an investment and the marginal profitability of the shock rises with the shock and the hazard rate is non-decreasing, expected profits are maximized for any given level of external finance through standard debt contracts. This occurs because competition intensity is inconsequential in low-profitability states and states where the marginal contribution of investment relative to the productivity shock is minimal, rendering firms as full residual claimants in high-profitability states and those where the marginal contribution of investment relative to the productivity shock is significant.

When determining the optimal debt level, firms weigh two potentially conflicting effects. One measures how the firm's debt, with competitors' investments held constant, influences the total firm value, including debt and equity. The other effect considers how changes in competitors' investments, with their debt held constant due to changes in the firm's debt, affect the total firm value. This latter effect represents the strategic-commitment effect of debt, the sign of which depends on the impact of the profitability shock on externalities' size, whether investments act as strategic complements or substitutes, and the firm's beliefs about competitors' cash constraints in the investment sub-game.

If the strategic-commitment effect of debt for firm i is non-negative, the firm borrows the necessary amount to be unconstrained in the investment sub-game. Conversely, if the strategic-commitment effect of debt is negative, the firm borrows an amount that leaves it cash-constrained in the investment sub-game. This borrowing amount could be zero if the initial cash is substantial or if the strategic-commitment effect is strongly negative. The firm opts for this constraint to mitigate post-investment competition intensity, as it lacks credible commitment to refrain from investing the optimal amount if it had sufficient cash at the investment time. Therefore, the firm opts to intentionally restrict itself to reduce post-investment competition intensity. Debt does not induce a business-stealing effect as it does not affect equilibrium payoffs when investments are held constant. Its impact on competitors' profits solely arises from its influence on equilibrium investments and externalities' size.[5]

A sufficient condition for a positive strategic-commitment effect is that, among any pair of firms, the equilibrium responses regarding investments to a firm's cash shock and the impact of the profitability shock on externalities share the same sign. This condition is satisfied when investments act as complements, and the influence of externalities on profits rises with the profitability shock. Similarly, it holds true when investments are strategic substitutes, diminishing with competitors' cash, and the effect of externalities on profits decreases with the profitability shock. Conversely, the strategic-commitment effect turns negative

[5] This aspect differentiates the choice of debt from decisions such as product quality or location selection, which have strategic implications beyond mere investment dynamics.

when the signs differ.

Determining theoretically the sign of the strategic-commitment effect of debt in most oligopoly games poses a challenge,, except in straightforward duopoly games like Cournot and Bertrand with differentiated goods and linear demand, Hotelling's line, and specific oligopolies such as the Salop circle and Cournot with homogeneous goods. These models are frequently employed to examine the interplay between product market competition and firms' financial structure, as evidenced in various studies such as Brander and Lewis (1986) and Chevalier and Scharfstein (1996).

This highlights the relevance of the empirical nature of this inquiry. Research by Bloom, Schankerman, and Van Reenen (2013) suggests the prevalence of positive externalities across various industries in the US economy. Further support for this idea comes from studies such as Qiu and Wan (2015), who found that firms experiencing greater technology externalities tend to maintain higher cash reserves, and Nguyen and Kecskes (2016), who identified a positive correlation between heightened technology externalities and leverage. Additionally, Bustamante and Frésard (2021) discovered a significant complementarity in investments among peers in product markets, a trend observed across a majority of sectors, especially pronounced in concentrated markets with diverse firms and smaller firms with less precise information. These empirical findings highlight the presence of positive externalities. However, discerning the sign of the strategic-commitment effect across different industries necessitates further empirical investigation. Such work should focus on estimating the sign of externalities and analyzing how firms' equilibrium investments respond to competitors' cash shocks, considering both demand and technology spillovers. Additionally, it's essential to examine how a firm's strategic commitment and the business-stealing effect of investments are influenced by competitors' cash shocks.

From a policy perspective, it is crucial to determine the industries or markets where the strategic-commitment effect is positive or negative. This insight aids in understanding whether firms face cash constraints and the underlying reasons behind these constraints. In cases where the strategic-commitment effect is positive, the optimal debt strategy involves firms raising external funds to avoid cash constraints in the investment sub-game. Conversely, in scenarios where the strategic-commitment effect is negative, being cash-constrained in the investment sub-game becomes an optimal strategy to mitigate competition. In such instances, policies aimed at enhancing firms' returns on investment, such as tax breaks, or implementing measures like a cash-in-hand policy, could be more effective than simply improving access to external financing through soft credit policies, as firms cannot commit to refraining from investing extra cash when they are cash-constrained in the investment sub-game.

The remainder of the paper is structured as follows: The next section provides a literature review, followed by the presentation of the model in Section 3. Section 4 focuses on deriving the equilibrium in the investment sub-game, characterizing best responses, and analyzing comparative statics related to cash shocks. Subsequently, Section 5 delineates firms' best responses concerning debt, characterizing them and establishing the existence of a sub-game perfect equilibrium. Finally, the paper concludes with closing remarks in the last section.

2 Literature Review

This paper is related to ideas from two distinct bodies of literature: one investigating the interplay between firms' financial structures and product market competition, and the other exploring the complex relationship between competition and innovation.

In the first one, the seminal papers are Brander and Lewis (1986) and Bolton and Scharfstein (1990). In the former, the conflict is due to the limited liability of equity-holders, which shields them from some of the downside risk. When there are two firms competing a-la-Cournot, firms have an incentive to take out debt and increase production to gain a strategic advantage over their industry rivals. The latter consists of straight debt leaving firms vulnerable to predatory actions of its product market rivals, leading to a negative equilibrium relation between the intensity of product market competition and leverage.

Kovenock and Phillips (1995), Kovenock and Phillips (1997), Maksimovic (1988), Brander and Lewis (1988), Showalter (1995), and Chevalier and Scharfstein (1996) study the same issue. Some of these papers arrive at different results due to different assumptions. For instance, Brander and Lewis (1986) and Maksimovic (1988) conclude that debt makes firms tougher in the product market game, while Showalter (1995), who considers Bertrand price competition with differentiated goods, shows that debt induces firms to raise prices relative to an oligopoly model with differentiated price competition without debt and therefore arrives at the opposite conclusion. Povel and Raith (2004) shows, within the Brander and Lewis's (1986) setting, that when contracts are endogenous, bankruptcy is costly, and production costs are explicitly taken into account, there is no room for the strategic use of debt as a device to soften product market competition.[6]

The evidence also points to an unambiguous effect (see, for instance, Chevalier (1995), Phillips (1995), and Kovenock and Phillips (1997)). Campello (2006) shows, using a model of strategic commitment through renegotiable-proof contracts, that there is a non-monotonic association between external (debt-like) financing and product market outcomes. He finds that moderate debt, on the margin, yields market share gains, and additional indebtedness leads to significant sales underperformance.

[6] Faure-Grimaud (2000) and Maurer (1999) also derive debt as an optimal contract in models of product-market competition with financial constraints.

Akdoğu and MacKay (2008) find evidence that firms in monopolistic industries exhibit lower investment-q sensitivity and are slower to invest than firms in competitive industries, but investment-q sensitivity and investment speed are highest in oligopolistic industries. Xede et al. (2023) study the effect of competition laws on external financing and investment decisions around the world. They find that the positive association between the stringency of competition laws and external financing and investment is stronger for financially constrained firms, poorly governed firms, and firms in countries with poorer investor protection and weaker legal enforcement. Thakor and Lo (2022) provide causal evidence supporting that as competition increases, R&D-intensive firms invest more relative to existing assets in place, carry more cash, and maintain less net debt.

Most of this literature focuses on risk-shifting incentives through debt to equity-holders and on the strategic consequences of such incentives on production decisions. In these articles, firms may have incentives to take out debt and increase production to gain a strategic advantage over their industry rivals as they assume that outputs are strategic substitutes. Typically, these models examine full-information environments in which either contracts are exogenous and cannot be renegotiated, and they rule out asymmetries in rivals' financial status (e.g., differential access to credit). This mechanism is quite different from ours. The role of credit constraints in this literature is to limit production capacity and to affect the risk of bankruptcy, while in our setting, firms can always produce the optimal quantity in the product market, credit constraints determine the efficiency of production and/or profitability of production, and risk-shifting plays a role only to determine the optimal financial structure. In our case, debt is used strategically to solve an ex-post commitment problem.

Within the literature regarding investments in innovation, the main theme is the impact of competition on innovation both at the theoretical and empirical level. Schumpeter (1942) was the first to argue that large firms with market power accelerate the rate of innovation (see, also, Loury (1979), Grossman and Helpman (1991), Aghion and Howitt (1992), Caballero and Jaffe (1993), Martin (1993)).[7] In contrast, Arrow (1962) shows that a monopoly that does not face competition for existing and new technologies has fewer incentives to invest in process innovation than a firm in a perfectly competitive industry (see, also, Porter (1990), and Baily, Gersbach, Scherer, and Lichtenberg (1995)).[8]

[7] There is a literature inspired by the seminal work of Hart (1983), focusing on managerial incentives, such as Schmidt (1997), Aghion, Dewatripont, and Rey (1997), and Aghion, Dewatripont, and Rey (1999), that provides rationales for a positive correlation between competition and managerial effort that can be associated with innovation. This literature hinges on an unusual assumption, which is that managers minimize innovation costs subject to the constraint that the firm does not go bankrupt, instead of the more common assumption of profit maximization.

[8] A firm that has a monopoly position in a market has a flow of profit that it enjoys if no innovation takes place. The monopolist can increase its profit by innovating, but it loses the profits from its old technology, while under competition there is a large return to innovate since under the old technology firms make zero profits.

Aghion and Bolton (1997), Aghion, Harris, Howitt, and Vickers (2001), Aghion, Bloom, Blundell, Griffith, and Howitt (2005), and Gilbert, Riis, and Riis (2018) show, using a leader-lagging firm model, that there is an inverted U-shaped relationship between competition intensity and industry-wide innovation attributing it to strategic substitutes and spillovers defined as the probability that a follower moves one step ahead of the leader without investing in innovation. Vives (2008) and López and Vives (2016), using a different setting, also argue in favor of an "inverted-U" relationship between competition and innovation.

Boone (2000, 2001), Athey and Schmutzler (2001), and Schmutzler (2013) explore the link between competition and innovation, suggesting that a rise in competitive pressure affects a firm's incentives based on its efficiency relative to competitors. More efficient firms benefit, while less efficient ones suffer, highlighting varied responses to competition (see, for instance,Gilbert (2006) and Schmutzler (2010) for reviews).

Lyandres and Palazzo (2016) show that firms' cash holdings depend positively on the intensity of competition in future product markets and their innovation efficiency. They also show that firms' cash holdings are negatively affected by their competitors' cash-holding choices. These effects are more important for relatively more constrained firms and when competition is expected to be more intense. Ma, Mello, and Wu (2020) also study the strategic role of cash in a two-stage competition model similar to the one by Lyandres and Palazzo (2016), showing that in equilibrium firms hold large amounts of cash in both concentrated and diffuse industries in order to take advantage of future investment opportunities. This happens because of capital market frictions that make outside financing less timely and therefore this might not be available at the time it is required.

Empirical evidence on competition and innovation yields mixed results, with studies like Baily et al. (1995), Blundell, Griffith, and van Reenen (1995), and Nickell (1996) providing support for a positive relationship between industry-wide innovation and competition, while Aghion et al. (2005) provide evidence in favor of an "inverted-U" relationship between them and Hashmi (2013) a negative one.

Beneito, Coscollá-Girona, Rochina-Barrachina, and Sanchis (2015) find that greater product substitutability and higher costs of entry lead to more process innovation, but less product innovation, whereas increases in market size increase both product and process innovation. Kretschmer, Miravete, and Pernias (2012) report that higher competitive pressure, as measured by the elimination of exclusive territories in the French automotive market, lowers process innovation and increases product innovation. Hashmi and Biesebroeck (2016) finds, in the automobile industry, that adding another firm would lower the rate of innovation in this industry, and this effect is magnified with higher quality entrants, but industry-wide innovation increases in most market structures.

Examining competition between AMD and Intel, Goettler and Gordon (2011) find that AMD's competition negatively affects innovation speed but enhances consumer welfare due to the offsetting positive impact on prices, outweighing lower quality. In the hard drive industry, Igami (2017) discover that despite robust preemptive motives and cost advantages, incumbents are hesitant to innovate due to cannibalization, indicating that the replacement effect surpasses the preemption effect. Similarly, Igami and Uetake (2019) observe plateau-shaped equilibrium relationships between competition and innovation in the hard disk drive industry.

In Kang's (2020) investigation of collusion cases in the U.S., findings reveal that colluding firms experienced a substantial 48% surge in patent filings and a 30% expansion in the scope of innovation through exploration of new technological domains. However, when competition was reinstated following the breakdown of collusion, the heightened and diversified innovation activities regressed to their previous levels. This stems from the fact that as competition intensity diminishes, firms witness a rise in revenue, leading to the accumulation of more cash for subsequent investment.

This literature highlights, at both the theoretical and empirical levels, an ambiguous relationship between innovation and competition. This association seems to be partially contingent on whether the focus is on process or product innovation. The majority of papers in this field rely on specific duopoly models, assuming innovations are substitutes with substantial externalities. Notably, none of the papers in this literature delve into the role of external financing, competition, and innovation within the same framework. This limitation hinders our ability to understand the potential strategic effect of external financing, which could, as shown in this paper, discourage firms from optimal investment. Our paper fills this gap by revealing the strategic impact of external financing on firms' optimal investment decisions taking into account the type and intensity of competition, as well as the nature and size of externalities.

3 The Model

3.1 The Game and Payoffs

Consider an industry with n firms indexed, $i \in \mathcal{I} \equiv \{1, \ldots, n\}$, each endowed with initial cash $A^i \in \Re_+$, engaged in the following game: At the first stage, firms simultaneously choose the financial structure and the amount to borrow and, in the second-stage, firms simultaneously choose how much to invest $\mu \equiv (\mu^1, \ldots, \mu^n) \in \Re^n_+$. For instance, μ^i could be the investment in process innovation such as a new product feature that enhances demand, the adoption of a new standard and/or software, the investment in infrastructure, information technology, ot artificial intelligence.[9] In the final stage, payoffs are realized and payments take places.

Firm i's profits are $\pi^i(\mu, \xi^i) : \Re^n_+ \rightarrow \Re$, where ξ^i is a idiosyncratic productivity shock realized after the investment decisions are made and before payments take place. This is distributed with CDF $F(\xi^i)$ with full support $v \equiv [\underline{\xi}, \bar{\xi}]$ and density $f(\xi^i)$, which is log-concave. Firm i's investment cost is μ^i. The market structure will be held constant; that is, the number of firms is exogenous, and factors that could trigger changes in the number of competitors such as mergers or entry are not considered.

The profit function must be understood as the equilibrium profits of a sub-game that takes place after firms observe μ, where firms engage in product market competition by choosing simultaneously either prices or quantities or any other strategic variable.[10] For this interpretation to be valid, we need to assume that there exists a Nash equilibrium selection in the product-market competition game that can be identified for every investment profile μ.[11]

We use sub-indices to denote partial derivatives. For instance, π^i_j denotes the derivative of firm i's profit with respect to firm j's investment and c^i_i denotes the derivative of firm i's investment cost with respect to its own investment.

Assumption 1. *For all $i \in \mathcal{I}$,*

i. For all $(\mu, \xi^i) \in \Re^n_+ \times \Xi$, $\pi^i(\mu, \xi^i) - \mu^i$ is twice-continuously differentiable and strictly quasi-concave in μ^i.

ii. For all $(\mu, \xi^i) \in \Re^n_+ \times \Xi$, $\lim_{\mu^i \to 0}\{\pi^i_i(\mu, \xi^i) - 1\} > 0$ and $\lim_{\mu^i \to \infty}\{\pi^i_i(\mu, \xi^i) - 1\} < 0$.

iii. For all $(\mu, \xi^i) \in \Re^n_+ \times \Xi$, $\pi^i_{\xi^i}(\mu, \xi^i) > 0$ and $\pi^i_{i\xi^i}(\mu, \xi^i) \geq 0$.

Part i. ensures that in the absence of credit constraints, best-responses are unique and continuous. Part ii. are standard Inada's type conditions needed to have positive and finite best-responses. Part iii. says that both profits as well as the marginal utility of own investment rise with the productivity shock.

The profit function allows for any pair of investments i and j to be either complements; that is, the higher investment by firm j, the higher firm i's marginal return to its investment, or substitutes; that is, the

[9] A radical innovation is not considered in our setting since that has a significant impact on the market structure and on the economic activity of firms in that market. The radical innovation could, for example, change the structure of the market, create new markets, render existing products obsolete or make competing firms unprofitable.

[10] See, Athey and Schmutzler (2001), Schmutzler (2013) and Boone (2000, 2001) for a similar reduced-form approach.

[11] The standard approach is to assume that there is a unique locally-stable equilibrium profile in the third-stage that depends smoothly on investment and parameters. For instance, Milgrom and Roberts (1990) show that this holds for Bertrand competition with differentiated goods and Amir (1996) shows that this is the case for Cournot with fixed marginal costs when the inverse of the demand function is log-concave. To get uniqueness, it is usually assumed that product-market payoffs, given μ, satisfy the well-known dominant diagonal condition.

higher investment by firm j, the lower firm i's marginal return to its investment.

From here onwards, we will say that firm i has positive externalites on firm j when an increase in its investment (weakly) rises, ceteris-paribus, firm j's profits; that is, $\pi^i_j \geq 0$, and has negative externalities when the opposite happens; that is, $\pi^i_j < 0$.[12] For any pair of investments i and j, externalities could be either positive or negative.

Externalities could arise from technology spillovers, knowledge sharing, and/or incomplete appropriability, which may increase/decrease the productivity of other firms operating in similar technology areas, or strategic spillovers, reflecting product-market interactions that create an indirect link between the investment decisions of firms through their anticipated impact on product market competition.

Hence, there are four different regimes in which each pair of firms i and j can find themselves. One where firm i and j's investments are complements and externalites positive and second one when they are negative, a third one when investments are substitutes and externalies negative and, a fourth one, when they are positive.

Firms can finance their investments with debt, equity, or a mix of both. Profits are contractible, and therefore financial contracts specify repayment functions $R^i(\pi^i) : \Re_+ \longrightarrow \Re_+$ in exchange for external funding D^i. We restrict the repayment function to be non-decreasing, continuous almost everywhere, and to satisfy limited liability; that is: (i) $R^i(\pi^i) \leq \pi^i$, meaning firm i cannot be required to pay more than the profits available to it; and (ii) $R^i(\pi^i) \geq 0$, indicating that the investors' liability is limited to their investment in the firm. Subject to these constraints, firm i will choose the contract that maximizes its profits, provided that investors' expected profits are non-negative.

We assume that contracts between investors and firms are chosen simultaneously, and thus financial contracts themselves cannot be used as strategic commitment devices.[13]

3.2 Product Market Competition, Externalities, and Complementarities

Let's assume that firms compete choosing prices and firm i's demand is $D^i(p;\mu,\xi^i)$, where $p = (p^1,\ldots,p^n)$, and the marginal cost of production is $c^i(\mu;\theta^i)$, where $\theta^i = (\theta^{i1},\ldots,\theta^{i1})$ is the vector of technology spillovers. Firm i solves the following problem $\max_{p^i\in\Re_+} E_{\xi^i}\pi^i(p;\mu,\xi^i)$, where $E_{\xi^i}\pi^i(p;\mu,\xi^i) \equiv (p^i - c^i(\mu;\theta^i))E_{\xi^i}D^i(p;\mu,\xi^i)$. Let's denote the equilibrium price by $p(\mu)$ and expected profits $E_{\xi^i}\pi^i(p(\mu);\mu,\xi^i)$

[12] Externalities have been discussed in works such as Bloom et al. (2013), López and Vives (2016) and d'Aspremont and Jacquemin (1988).

[13] Alternatively, we could assume that they are not observable to other investors and firms.

evaluated at the equilibrium price profile for each investment profile μ by $\Pi^i(\mu)$. Hence,

$$\Pi_i^i = \underbrace{\sum_{j \neq i} (p^i - c^i) E_{\xi^i} D_{p^j}^i p_i^j}_{\text{Strategic−commitment Effect}} - \underbrace{c_i^i E_{\xi^i} D^i}_{\text{Efficiency Effect}} + \underbrace{(p^i - c^i) E_{\xi^i} D_i^i}_{\text{Business−stealing Effect}},$$

An increase in firm i's investment affects its profits through three channels: The strategic-commitment effect of investment, whose sign is determined by how firms compete in the product market and technology spillovers since equilibrium prices depend on θ; the efficiency effect due to lower marginal costs, which is larger when firm i's demand is higher; and the business-stealing effect that occurs when the investment, holding prices constant, increases demand.

Because D^i rises with p^j when goods are gross substitutes, the sign of the strategic-commitment effect is determined by how prices respond to firm i's investment. If $E_{\xi^i} \pi^i(p; \mu, \xi^i)$ is supermodular in p and either spillovers are positive when investments affect marginal costs or $E_{\xi^i} D^i(p; \mu, \xi^i)$ is log-convex in (p, μ) when investments modify demand or both, then prices rise with μ. Otherwise, some of them can raise and some can fall, making it impossible to pin down equilibrium responses without making more assumptions.

Firm i and firm h's investment are complements when $\Pi_{ih}^i > 0$. This amounts to understanding how μ^h affects the strategic-commitment effect of investments, the efficiency, and the business-stealing effect.

A hike in μ^h affects these through several different channels that tend to result in opposing forces. The first one is the impact on firm i's profit margin. Holding prices constant, an increase in μ^h rises profit margins when technology spillovers are positive and decreases them when they are negative. Holding marginal costs constant, profit margins increase when equilibrium firm i's price increases with μ^h. This, for instance, happens when profits are supermodular in p and investment spillovers are positive.

The second one is how μ^h changes the intensity of gross substitution. This depends again on how equilibrium prices respond to μ and whether $E_{\xi^i} D_{p^j}^i$ rises or falls with p.

The third one is the impact on c_i^i, which should be negative when spillovers are positive and positive otherwise.

The fourth one is the impact on demand, which, holding prices constant, is positive when spillovers are positive and negative otherwise, and if prices increase, demand increases when goods are gross substitutes.

The last one is the impact on $E_{\xi^i} D_i^i$, which depends again on how equilibrium prices change and the direct impact.

At this level of generality, it is impossible to sign all these effects, but in the case of linear demand and when marginal costs are linear in μ, this is possible since the second derivatives vanish. In fact, Proposition

1 in Vives (2009) provides conditions for $\Pi(\mu)$ to be supermodular in μ. Beside from standard regularity conditions, $E_{\xi^i}\pi^i(p,\mu;\xi^i)$ must be supermodular in (p,μ), which immediately rules out product-market games with strategic substitutes such as Cournot, must have convex spillovers in p, and prices have to be supermodular in μ. Hence, in most cases, $E_{\xi^i}\pi^i(\mu,\xi^i)$ is not supermodular in μ. Still, we cannot rule out this possibility since it happens in specific commonly used models such as Cournot and Bertrand duopoly with differentiated goods when spillovers are sufficiently large relative to the degree of differentiation (the competition intensity parameter).

The sign of the externalities is determined by the sign of the following expression,

$$\Pi_h^i = \underbrace{\sum_{j\neq i}(p^i - c^i)E_{\xi^i}D^i_{p^j}p^j_h}_{\text{Strategic}-\text{spillover Effect}} - \underbrace{c^i_h E_{\xi^i}D^i}_{\text{Technology}-\text{spillover Effect}} + \underbrace{(p^i - c^i)E_{\xi^i}D^i_h}_{\text{Demand}-\text{spillover Effect}} .$$

An increase in firm h's investment affects firm i's profits in three distinct ways.

First, through the strategic-spillover effect of investment, whose sign is determined by how firms compete in the product market and spillovers, given that equilibrium prices depend on θ.

Second, the technology-spillover effect due to the impact of competitors on firm i's marginal costs, which is positive when spillovers are positive and negative otherwise. When spillovers are positive, this effect is more pronounced when firm i's demand is higher.

Third, the demand-spillover effect that arises when competitors' investment modifies firm i's demand. For instance, this effect is negative when firm h introduces a new product feature and could be positive when a firm finds a new use for a product produced by competitors, as is common in the pharmaceutical industry. Because equilibrium prices depend on spillovers, this effect also hinges on θ. As before, if $E_{\xi^i}\pi^i(p;\mu,\xi^i)$ is supermodular in p and either spillovers are positive when investments affect marginal costs or $E_{\xi^i}D^i(p;\mu,\xi^i)$ is log-convex in (p,μ) when investments modify demand or both, then prices rise with μ. Otherwise, some prices may increase while others fall, making it challenging to determine equilibrium responses.

Thus, for $E_{\xi^i}\pi^i(\mu,\xi^i)$ to be able to speak to an ample variety of oligopoly models and parameters regarding primitives such as preferences, spillovers, etc., it is essential to allow for both positive and negative externalities as well as complements and substitute investments, as these are determined by how firms compete in the product market, the type of investment, and technology spillovers.

4 Investment Sub-game

4.1 Best-Responses

When competing firms' investment profile is μ^{-i} and firm i has an amount of cash equal to $K^i \equiv A^i + D^i$, firm i solves the following problem

$$\max_{\mu^i \in [0,K^i]} \Big\{ \int_{\xi^i \in \Xi} \big(\pi^i(\mu,\xi^i) - R^i(\pi^i(\mu,\xi^i))\big) dF(\xi^i) + K^i - \mu^i \Big\}.$$

From here onwards, we assume the following

Assumption 2.

$$\Pi^i(\mu, R^i) \equiv \int_{\xi^i \in \Xi} \big(\pi^i(\mu,\xi^i) - R^i(\pi^i(\mu,\xi^i))\big) dF(\xi^i)$$

is strictly quasi-concave in μ^i for all μ^{-i}.

Because $\pi^i(\mu,\xi^i)$ is strictly quasi-concave in μ^i, a sufficient condition for this to hold is that $\pi^i(\mu,\xi^i) - R^i(\pi^i(\mu,\xi^i))$ is non-decreasing in $\pi^i(\mu,\xi^i)$. This is the case in a standard debt contract as well as for any equity financing contract or a combination of both of them.

The first-order condition is given by:

$$\int_{\xi^i \in \Xi} \big(1 - R^i_i(\pi^i(\mu,\xi^i))\big)\pi^i_i(\mu,\xi^i) dF(\xi^i) - 1 - \lambda^i = 0,$$

where $\lambda^i \geq 0$ is the Langrange's multiplier for the cash constraint: $\mu^i \leq K^i$.

Let $BR^{iu}(\mu^{-i})$ be the solution to the first-order condition when $\lambda^i = 0$. This exists, is unique and interior.[14] Observe that the best-response function when the firm is unconstrained; i.e., $BR^{iu}(\mu^{-i})$, does not depend on K^i, but it does depend on the financial contract. In fact, it falls with the slope of the repayment function R^i. Conversely, the best-response function when the firm is constrained; i.e., K^i, depends only on K^i and not on R^i. Hence, firm i's best response is the investment level that maximizes expected profits when the firm is not cash-constrained and it is the highest investment allowed by the available cash otherwise. Thus,

$$BR^i(\mu^{-i}; K^i) \equiv \begin{cases} BR^{iu}(\mu^{-i}) & \text{if } BR^{iu}(\mu^{-i}) \leq K^i, \\ K^i & \text{if } BR^{iu}(\mu^{-i}) > K^i. \end{cases} \tag{1}$$

[14]This follows from strict the quasi-concavity of profits, the fact that the constraint set is convex-valued, the limited-liability constraint that imposes that $R^i_i(\pi^i) \in [0,1]$ for all π^i, and the Inada's type conditions.

This is continuous and piece-wise differentiable with a kink at the point where $BR^{iu}(\mu^{-i}) = K^i$.[15]

Let $\mu^{i1} \equiv BR^{iu1}(\mu^{-i}; K^i)$ be the optimal investment profile when the contract is R^{i1} and $\mu^{i0} \equiv BR^{iu0}(\mu^{-i}; K^i)$ be the optimal investment profile when the contract is R^{i0}.[16]

Proposition 1. *Suppose two different financial contracts R^{i0} and R^{i1} such that $\int_{\xi^i \in \Xi} R^{i0}(\pi(\mu, \xi^i))dF(\xi^i) = \int_{\xi^i \in \Xi} R^{i1}(\pi(\mu, \xi^i))dF(\xi^i)$, for which there exists a profitability shock $\hat{\xi}^i$ such that $R_i^{i1} \geq R_i^{i0}$ for all $\xi^i \leq \hat{\xi}^i$ and $R_i^{i1} \leq R_i^{i0}$ for all $\xi^i \geq \hat{\xi}^i$. Then, for any μ^{-i}, $BR^{iu0}(\mu^{-i}) \leq BR^{iu1}(\mu^{-i})$ and $\Pi^i(\mu^{i0}, \mu^{-i}, R^{i1}) - \mu^{i0} \leq \Pi^i(\mu^{i1}, \mu^{-i}, R^{i0}) - \mu^{i1}$.*

This implies that firms faced with a standard debt contract $\min\{R^i, \pi^i\}$ invest more, ceteris-paribus competitors' investments, than when faced with equity financing that yields the same expected repayment and under both contracts they are not cash-constrained. This stems from the fact that firms are able to appropriate a larger share of firms' marginal returns to the investment under a debt contract than under equity financing when marginal returns are larger since $\pi^i_{i\xi^i} \geq 0$.

In other words, debt makes the firm full residual claimant on states where the investment is marginally more profitable, while equity makes the firm a partial residual claimant in every possible state. Because the firm under contract R^{i1} will invest more than under contract R^{i0}, we have the following corollary.

Corollary 1. *Suppose two different financial contracts R^{i0} and R^{i1} such that $E_{\xi^i} R^{i0}(\pi(\mu, \xi^i)) = E_{\xi^i} R^{i1}(\pi(\mu, \xi^i))$, for which there exists a profitability shock $\hat{\xi}^i$ such that $R_i^{i1} \geq R_i^{i0}$ for all $\xi^i \leq \hat{\xi}^i$ and $R_i^{i1} \leq R_i^{i0}$ for all $\xi^i \geq \hat{\xi}^i$. Then, for any μ^{-i}, firm i is cash-constrained under contract R^{i1} but not under contract R^{i0} for all $K^i \in (BR^{iu0}(\mu^{-i}; K^i), BR^{iu1}(\mu^{-i}; K^i))$.*

Observe that investments are strategic complements when

$$\int_{\xi^i \in \Xi} \big((1 - R_i^i(\pi^i(\mu, \xi^i)))\pi^i_{ij}(\mu, \xi^i) - R^i_{ii}(\pi^i(\mu, \xi^i))\pi^i_j(\mu, \xi^i) \big) dF(\xi^i) \geq 0.$$

Whether investments are complements or substitutes depends not only on whether the gross marginal return to the own investment rises or falls with competitors' investment but also on how they affect firms profits (externalities) as well as the shape of the repayment function. For instance, if π^i is supermodular in μ, externalities are positive, and the repayment function is concave, investments are strategic complements. In contrast, if π^i is submodular in μ, externalities are non-positive, and the repayment function is concave,

[15] Continuity follows from the Maximum Theorem, the fact that the objective function is strictly quasi-concave and the constraint set is compact and convex-valued.

[16] Proofs can be found in the Appendix.

investments are strategic substitutes. Hence, when firms are not cash-constrained, external financing as well as the type of financing are crucial determinants of the strategic relationship between investments.

The example below shows this by means of comparing a standard debt contract with limited liability and equity financing.

Example 1. *Let the debt contract be* $R^i(\pi) = \min\{R^i, \pi^i\}$ *and* $\xi^i(R^i, \mu)$ *be the lowest shock such that* $R^i = \pi^i(\mu, \xi^i)$*. Then, investments are strategic complements whenever*

$$\int_{\xi^i(R^i,\mu)}^{\bar{\xi}^i} \pi^i_{ij}(\mu, \xi^i) dF(\xi^i) - \pi^i_i(\mu, \xi^i) \frac{\pi^i_{\xi^i}(\mu, \xi^i)}{\pi^i_j(\mu, \xi^i)} f(\xi^i)|_{\xi^i(R^i,\mu)} \geq 0.$$

where $\pi^i_{\xi^i}(\mu, \xi^i) > 0$.

Now, let $R^i(\cdot)$ *be equity financing where* $1 - \alpha^i$ *is the share of the return paid to outside equity. Then, investments are strategic complements whenever*

$$\alpha^i \int_{\xi^i \in \Xi} \pi^i_{ij}(\mu, \xi^i) dF(\xi^i) \geq 0.$$

Let's define $\mu^j(\mu^{-ij}, K^i)$ as the solution to $BR^{iu}(\mu^j, \mu^{-ij}) = K^i$. When $BR^{iu}(\mu^j, \mu^{-ij})$ rises with μ^j, and $\mu^j > \mu^j(\mu^{-ij}, K^i)$, firm i is cash-constrained, while when $BR^{iu}(\mu^j, \mu^{-ij})$ falls with μ^j, and $\mu^j < \mu^j(\mu^{-ij}, K^i)$, firm i is cash-constrained. Hence, we have the following result.

Lemma 1. *For any* $\mu^{-ij} \in \Re^{n-2}_+$.

i. If investments are complements and $\lim_{\mu^j \to 0}(\Pi^i_i(\mu, R^i) - 1) \leq 0$,[17] *then there exits a unique* $\mu^j(\mu^{-ij}, K^i) \geq 0$ *such that*

$$\frac{\partial BR^i(\mu^{-i}; K^i)}{\partial \mu^j} = \begin{cases} -\frac{\Pi^i_{ij}(\mu, R^i)}{\Pi^i_{ii}(\mu, R^i)}|_{BR^{iu}(\mu^{-i})} & \text{if } \mu^j < \mu^j(\mu^{-ij}, K^i), \\ 0 & \text{if } \mu^j \geq \mu^j(\mu^{-ij}, K^i). \end{cases} \tag{2}$$

ii. If investments are substitutes and $\lim_{\mu^j \to 0}(\Pi^i_i(\mu, R^i) - 1) \geq 0$*, then there exits a unique* $\mu^j(\mu^{-ij}, K^i) \geq 0$ *such that*

$$\frac{\partial BR^i(\mu^{-i}; K^i)}{\partial \mu^j} = \begin{cases} -\frac{\Pi^i_{ij}(\mu, R^i)}{\Pi^i_{ii}(\mu, R^i)}|_{BR^{iu}(\mu^{-i})} & \text{if } \mu^j \geq \mu^j(\mu^{-ij}, K^i), \\ 0 & \text{if } \mu^j < \mu^j(\mu^{-ij}, K^i). \end{cases} \tag{3}$$

[17] The existence and uniqueness follows from the fact that $\Pi^i_i(\mu, R^i) - 1$ is monotonically increasing in μ^j, non-positive at $\mu^j = 0$, and the intermediate-value theorem.

When firm i is not cash-constrained, its best response increases with μ^j when their investments are complements and falls with it when they are substitutes. When firm i is strictly cash-constrained, its investment is independent of firm j's investment. When firm i's cash is such that it is exactly able to finance its unconstrained investment, its best response reacts differently to an increase in firm j's investment : when investments are complements, if μ^j rises, firm i's investment stays the same since it wishes to invest more but it does not have the cash to do so, while when investment are substitutes, it reduces its investment since it is feasible. In contrast, when firm j lowers its investment, firm i lowers its investments when they are complements and keeps the investment unchanged when they are substitutes since it would like to increase it but it lacks the cash to do so.

Because an increase in firm j's available cash has no direct impact on firm i's best-response function, a hike in firm j's cash has no effect on firm i's best response when firm j is unconstrained since it does not change its own investment. In contrast, when firm j is cash-constrained, the change in firm i's best-response function with firm j's cash is given $\frac{\partial BR^i(\mu^{-i};K^i)}{\partial K^j} = \frac{\partial BR^i(\mu^{-i};K^i)}{\partial \mu^j}$ since $\mu^j = K^j$.

Because an increase in firm i's available cash has no direct impact on firm j's best-response function, an increase in K^i rises its set of feasible investments, and does not change its payoff, firm i's best response increases with K^i when firm i is cash-constrained and is independent of it when it is unconstrained. When K^i is such that firm i's best-response when unconstrained satisfies the cash constraint with equality, firm i's best-response is independent of K^i when K^i increases since K^i has no impact on marginal returns and the unconstrained investment remains feasible, while a decrease in K^i lowers the best response since $BR^{iu}(\mu^{-i})$ is no longer feasible.

Last but not least, competitors' investment play a role on whether firm i is cash-constrained or not since the unconstrained investment depends on μ^{-i}. When investments are complements, an increase in a competitor's investment makes an unconstrained inframarginal firm cash-constrained since the firm wishes to invest more, while when investments are substitutes, an increase in a competitor's investment makes a constrained inframarginal firm unconstrained since the firm wants to decrease its investment.

4.2 Nash Equilibria in the Sub-Investment Game

Because for each $i \in \mathcal{I}$, firm i's profits are strictly quasi-concave in μ^i, continuous in μ^{-i}, and $\mu^i \in [0, K^i]$ is compact and convex, Debreu-Glicksberg-Fan's Theorem implies the following.

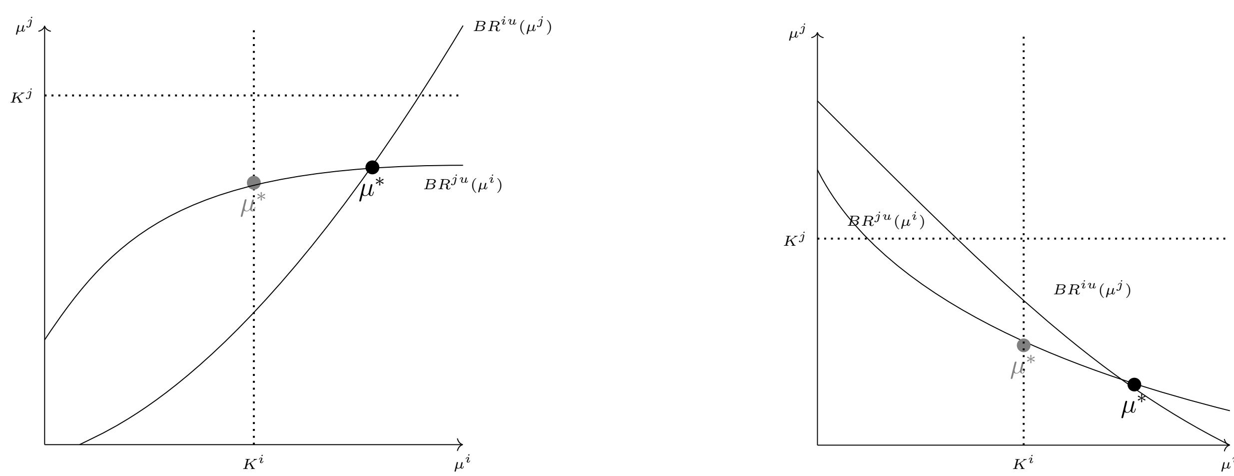
Proposition 2. *There exits a Nash equilibrium in the investment sub-game, denoted by $\mu(K)$.*[18]

[18]The argument will be omitted when there is no risk of confusion.

When firm i is cash-constrained, the equilibrium investment satisfies $\mu^i = K^i$, while when it is unconstrained, $\mu^i(K)$ equals to the unique solution to the first-order condition: $\Pi_i^i(\mu^i, \mu^{-i}(K), R^i) - 1 = 0$. Hence, neither firm i's competitors nor firm i use cash strategically to soften competition in the investment sub-game, since each firm is full residual claimant on its ex-post cash regardless of the cash level and repayment. This, however, does not preclude firms from using the choice of the financial contract to stifle competition.

4.3 Understanding the relationship between Investments, Industry-Wide Investments, and Cash

In this sub-section, we study the relationship between a firm's cash and the equilibrium investment at the firm and industry-wide level. Figure 1 depicts best-response functions and the equilibrium for a duopoly. Panel 1(a) presents the case in which investments are complements, a unique equilibrium when no firm is constrained (black dot), and a unique equilibrium when firm i is cash-constrained and firm j is not (gray dot). Panel 1(b) shows the case in which investments are substitutes, there is a unique equilibrium when no firm is constrained (black dot), and a unique equilibrium when firm i is cash-constrained and firm j is not (gray dot).



(a) Complements **(b)** Substitutes

Fig. 1. Nash Equilibria in the Investment Sub-game

Panel 1(a) shows that the fact that firm i is cash-constrained and firm j is not results in both firms investing less than when none is constrained. When only firm i is cash-constrained, an increase in firm i's cash increases firm j's investment. If both firms were to be constrained, then the increase in firm i's cash will increases its investment, it will not rise firm j's investment. Hence, when investments are complements, a positive cash shock to a constrained firm never decreases competitors' investment and rises industry-wide investment.

Panel 1(b) shows that when firm i is cash-constrained, its investment is lower but firm j's investment is higher when compared to the situation where firm is not constrained. Hence, a-priori industry-wide investment could either increase or decrease. If it happens that firm j is also constrained and firm i's cash rises, firm j's investment stays constant up to the point where $BR^{uj}(K^i) = K^j$ and beyond that its investment falls since its best response is negatively sloped. So, first industry-wide investment rises and then may either rise or fall depending on how sensitive is firm j's best response to firm i's investment.

In the case of complementary investments, the graphical analysis holds for any number of firms since best-response functions are non-decreasing in competitors' investments and in K. Thus, the monotone comparative static results in Milgrom and Roberts (1990) imply the following.

Proposition 3. *Suppose investments are complements and $\mu(K)$ is an extremal equilibrium. Then, for any $K' \geq K$ and $K^{j'} > K^j$ for some j, $\sum_{i\in\mathcal{I}} \mu^i(K') \geq \sum_{i\in\mathcal{I}} \mu^i(K)$ and $\mu^i(K') \geq \mu^i(K)$ for all $i \in \mathcal{I}$.*

When investments are substitutes, the graphical analysis carried out above cannot be extended to more than two firms. In fact, in a parameterized game of strategic substitutes, two counterweighing forces come into play when firm i increases its cash: a direct effect that results, holding competitors' investments fixed, in an increase in firm i's investment when it is cash-constrained; and an indirect effect that works as follows: an increase in firm i's investment puts pressure on any firm $j \neq i$ to decrease investment, which may in turn affect firm $l \neq j$ differently. Whether firm l is more sensitive to the initial increase in firm i's investment or to the decrease in firm j's investment depends on how sensitive the marginal return of each firm is to its competitors' investment. Therefore, a priori, we cannot rule out that some firms increase their investments while others decrease them.

Since an increase in firm i's cash does not change competitors' best-response functions, it must be the case that at least one firm lowers its investment, and thus, increasing industry-wide investment requires that the direct effect dominates the indirect effects. This implies that monotone comparative statics results, such as those in Roy and Sabarwal (2010), are not applicable in our setting, as an increase in firm i's cash leaves competitors' best-response functions unchanged, and these functions are strictly decreasing for unconstrained firms and non-increasing for constrained ones.[19]

[19] Athey and Schmutzler (2001) also deal with strategic substitutes and robust comparative statics to study increasing dominance. Their results assume exchangeability with respect to investment and firm's competitiveness given by pre-investment marginal costs, and conditional uniqueness. This means that if there are two different firms i and j, conditional on that there are two equilibria in which firms other than firms i and j invest the same, the equilibrium must be unique. The exchangeability assumption requires in general that the cross-price effects are identical for all firms. Furthermore, they assume that an increase in any firm's competitiveness

Let's define $\mathcal{I}^u(K) \equiv \{i \in \mathcal{I} | \Pi_i^i(\mu(K), R^i) - 1 = 0\}$. $\mathcal{I}^u(K)$ is the set of firms that are unconstrained in the investment sub-game equilibrium and therefore their investments are determined by unconstrained profit maximization. In order to carry out comparative statics using the implicit function theorem, we need to assume that an infinitesimal change in any parameter neither causes a firm to go from being cash-constrained to not being constrained nor vice versa. This requires that there is no firm for which $BR^{iu}(\mu^{-i}(K)) = K^i$, because at this point the equilibrium condition is not differentiable and thus the Implicit Function theorem (IFT) cannot be applied.[20] We will assume this is the case from henceforwards.

Christensen (2018) shows that when the negative of the Jacobian of the first-order conditions is a B_0-matrix, which is a generalization of the well-known and widely used dominant-diagonal condition, the sum of equilibrium strategies increases with any parameter that (weakly) rises each firm's best-response function.[21]

The negative of the Jacobian of the first-order conditions, denoted by Π, is a B_0-matrix if for all $i \in \mathcal{I}$,

$$\Big(\Pi_{ii}^i + \sum_{j \neq i} \Pi_{ij}^i\Big)|_{\mu=\mu(K)} \leq n \min_{j \in \mathcal{I}} \{0, \Pi_{ij}^i | j \neq i\}|_{\mu=\mu(K)}. \tag{4}$$

This says that the average effect on firm i's marginal return to investment of an equal size increase in each firm's investment is non-positive and less than the effect of an increase in any single player j's investment on player i's ($j \neq i$) marginal return. We will call this mean dominance for short.

Proposition 4. *Suppose that $\mathcal{I}^u(K)$ does not change with K^j and Π is a B_0-matrix, then $\sum_{i \in \mathcal{I}} \mu_{K^j}^i(K) \geq 0$ for all $j \in \mathcal{I}$.*

This establishes that an increase in firm i's cash (weakly) raises industry-wide investment regardless of whether investments are complements or substitutes. This stems from the fact that for a cash-constrained firm, an increase in its cash raises its best-response function, while competitors' best-response functions remain unchanged. Mean dominance ensures that firm i's partial effect dominates the sum of indirect effects due to competitors' optimal responses and is higher than n times the effect of an increase in any single firm

increases its marginal return to investment and decreases competitors' marginal returns. The first two assumptions makes the n player game behave as if it were a 2 player game, which is the same that happens in aggregative games. This allows them to use the results coming from robust monotone comparative statics. Given these assumptions, they are able to prove that in equilibrium, a leader invests more than a laggard. We cannot apply this technique here because best-response functions do not change with a shock to a competitor's cash.

[20] There are implicit function theorems for functions that have kinks that could be applied here, but will make the analysis unnecessarily complex and no new insight will be gained.

[21] See, Peña (2001) for general properties of B-matrices. B_0-matrices have non-negative determinants, the principal submatrices of a B_0-matrix are also B_0-matrices and as such have non-negative determinants too, they have non-negative principal minors, and the column sum of the inverse of a B_0-matrix is non-negative. The row sum of the inverse of a matrix is non-negative when the transpose of the matrix is a B_0-matrix.

j's marginal change. This, together with the fact that no best-response function decreases with K^j, implies that industry-wide investment increases. Hence, an exogenous positive shock to any cash-constrained firm's cash will result in an increase in its own investment, a non-decrease in industry-wide investment, and the decrease of the investment of at least one firm unless all firms are constrained.

Mean dominance places a limit on how heterogeneous firms could be in terms of their best-response functions; that is, $\Pi^i ij$ cannot be too different from $\Pi^i il$ for each i, j, l with $i \neq j \neq l$, but it does not require a particular type of complementarity nor symmetric firms. The advantage of this approach is that it allows dealing with: i) different firms having different complementarity relationships; ii) firm heterogeneity, which is fundamental when dealing with credit constraints since it allows some firms to be constrained and some not, different efficiency levels, different spillovers, and, more importantly, different financial contracts; and (iii) more than two firms. The main disadvantage is that the result is not based entirely on robust conditions on fundamentals and relies on local conditions that make use of fundamentals as well as equilibrium conditions. Hence, the result is local in nature.[22]

5 The Sub-game Perfect Equilibrium

5.1 Financial Contract Choices

Because we abstract from bankruptcy costs and from the tax advantages of debt, issuing debt is strictly a break-even transaction for the firm, except for the fact that equilibrium investment levels will depend on debt levels. If, by way of contrast, investment levels were exogenously fixed then issuing debt would be purely neutral, having no effect on total value, as in a Modigliani-Miller world.

Because the capital market is perfectly competitive, financiers make zero expected profits and therefore D^i must be equal to the expected repayment. Hence, conditional on competitors choosing K^{-i}, each firm i

[22] In the oligopoly literature with differentiable payoffs one typically places restrictions on the off-diagonal terms of the Jacobian relative to the terms along the main diagonal. We do the same, yet our results are more general than those that could be derived using known results that make use of the IFT since asking for a Jacobian to be a B_0 matrix is weaker than Dixit's (1986) dominant diagonal property, and Corchon's (1994) condition, and generalizes the aggregation results in Acemoglu and Kaae Jensenz (2009), since their results do not apply to oligopolies with differentiated goods and investment models with externalities (see, Christensen (2018)). See, also, Jinji (2014) for different conditions.

solves the following problem[23]

$$\max_{(K^i,R^i)\in\Re_+\times\mathcal{R}} \left\{ \int_{\xi^i\in\Xi} \big(\pi^i(\mu(K),\xi^i) - R^i(\pi^i(\mu(K),\xi^i))\big)dF(\xi^i) + K^i - \mu^i(K)\right\}$$

$$\text{subject to} \tag{5}$$

$$K^i = A^i + \int_{\xi^i\in\Xi} R^i(\pi^i(\mu(K),\xi^i))dF(\xi^i),$$

$$0 \leq R^i(\pi^i(\mu(K),\xi^i)) \leq \pi^i(\mu(K),\xi^i),\ \forall\,\xi^i,$$

$$R^i_i(\pi^i(\mu(K),\xi^i)) \geq 0,\ \forall\,\xi^i.$$

From the perspective of the firm, contracts that yield the same expected repayment and thus result in the same external funding amount are considered identical when that amount falls within the range of $A^i + D^i \leq BR^{iu}(\mu^{-i}(K))$. In such cases, firm i will invest $A^i + D^i$ regardless of the specific contract terms. However, when the funding exceeds $BR^{iu}(\mu^{-i}(K))$, the situation changes. Proposition 1 demonstrates that certain contracts provide stronger investment incentives despite offering the same expected repayment. Therefore, it's crucial to recognize that optimal investment depends not only on K but also on $R \equiv (R^1,\ldots,R^n)$.

Because the expected repayment equals the amount of external finance raised, and firms cannot commit to not investing the available cash when it is lower than the unconstrained optimal investment or to overinvest beyond this level, there is no strategic use of cash in the investment sub-game. Additionally, there is no benefit from raising more external funds than needed to finance the optimal investment. Thus, $D^i = \max 0, \mu^i(K) - A^i$. In what follows, we will focus on the case where $BR^{iu}(K) \geq A^i$. Then, after substituting this into the objective function and using integration by parts, firm i's optimization problem is as follows:

$$\max_{R^i\in\mathcal{R}} \int_{\xi^i\in\Xi} \pi^i_{\xi^i}(\mu(K),\xi^i)(1-F(\xi^i))d\xi^i + \pi^i(\mu(K),\underline{\xi}^i)$$

$$\text{subject to} \tag{6}$$

$$0 \leq R^i(\pi^i(\mu(K),\xi^i)) \leq \pi^i(\mu(K),\xi^i),\ \forall\,\xi^i,$$

$$R^i_i(\pi^i(\mu(K),\xi^i))\pi^i_{\xi^i}(\mu(K),\xi^i) \geq 0,\ \text{a.e.},$$

where $K^i \equiv A^i + \int_{\xi^i\in\Xi} R^i_i(\pi^i(\mu(K),\xi^i))\pi^i_{\xi^i}(\mu(K),\xi^i)(1-F(\xi^i))d\xi^i + R^i(\pi^i(\mu(K),\underline{\xi}^i)$.

In what follows, we adopt the following assumption

[23]Observe that choosing debt is different from choosing another strategic variable since debt has no direct impact on firms' payoffs, it affects them only through the equilibrium strategies in the investment sub-game. Hence, debt neither results in a business-stealing effect nor in an efficiency effect.

Assumption 3. *For all $\mu \in \Re^n_+$, $\frac{\pi^i_i(\mu,\xi^i)}{\pi^i_{\xi^i}(\mu,\xi^i)}$ is increasing in ξ^i.*

A sufficient condition for the first part is that $\pi^i_{\xi^i}(\mu, \xi^i)$ is non-increasing in ξ^i. This condition implies that the ratio between the marginal contribution of the investment to profits and the marginal contribution of the profitability shock is higher as the state becomes higher. In other words, the relative marginal contribution of the investment to profits is greater in states with higher shocks.

Proposition 5. *Suppose that assumption 3 holds. Then, there exits a threshold $\xi^i(D^i) \in [\underline{\xi}^i, \bar{\xi}^i)$ such that the optimal contract that implements external financing D^i and the investment profile $\mu(K)$ is the standard debt contract*

$$R^i(\pi^i(\mu(K), \xi^i)) = \begin{cases} \pi^i(\mu(K), \xi^i) & \text{if } \xi^i \in [\underline{\xi}^i, \xi^i(D^i)], \\ \pi^i(\mu(K), \xi^i(D^i)) & \text{if } \xi^i \in [\xi^i(D^i), \bar{\xi}^i]. \end{cases} \tag{7}$$

The optimal financial contract maximizing profits, given the need to raise external funds equal to D^i, is a standard debt contract. This optimality arises from risk neutrality, a non-decreasing hazard rate, and the observation that $\pi^i_i/\pi^i_{\xi^i}$ increases with ξ^i. Consequently, for a given investment profile, the firm prefers a contract where it retains full residual claimancy in states where the marginal contribution of investment relative to profitability shock is higher. Thus, the presence of market power in investments challenges the Modigliani-Miller theorem, as the optimal financial structure involves debt, indicating the dependence of real decisions on financial contracts. Capital suppliers are incentivized to participate, as they receive expected income equal to the opportunity cost of their investment, ensuring active engagement in the financial structure.

The firm's objective in choosing debt is to maximize total firm value, which comprises equity and debt value. If potential debtholders are foresighted, owners can issue bonds promising to pay D^i only for their true value, considering the possibility of bankruptcy. Hence, firm i selects D^i to solve the following problem:

$$\max_{D^i \in \Re_+} \Big\{ \int_{\underline{\xi}^i}^{\bar{\xi}^i} \pi^i(\mu(K), \xi^i) dF(\xi^i) + A^i - \mu^i(K) \Big\}.$$

The first-order condition is given by

$$\sum_{j=1}^{n} \int_{\underline{\xi}^i}^{\bar{\xi}^i} \pi^i_j(\mu(K), \xi^i) \mu^j_{K^i}(K) dF(\xi^i) - \mu^i_{K^i}(K) \leq 0.$$

Using the envelope theorem and integrating-by-parts, the first-order condition can be written as follows

$$\left(\int_{\xi^i(D^i)}^{\bar{\xi^i}} \pi^i_{i\xi^i}(\mu(K),\xi^i)(1-F(\xi^i))d\xi^i + \int_{\underline{\xi}^i}^{\xi^i(D^i)} \pi^i_{i\xi^i}(\mu(K),\xi^i)(1-F(\xi^i))d\xi^i - 1\right)\mu^i_{K^i}(K) + \sum_{j\neq=i}\mu^j_{K^i}(K)\int_{\underline{\xi}^i}^{\bar{\xi^i}} \pi^i_{j\xi^i}(\mu(K),\xi^i)(1-F(\xi^i))d\xi^i \leq 0. \tag{8}$$

The sum of the first two terms represents the impact of debt on the total value of the firm, comprising debt and equity value, multiplied by how firm i's debt increases its investment. The third term denotes the strategic-commitment effect of debt. Thus, firm i must weigh the benefit of being unconstrainedâĂŤnamely, the increase in total firm value while competitors' investments remain constantâĂŤagainst the change in total firm value due to competitors' optimal responses to an increase in firm i's debt, i.e., the strategic-commitment effect.

The first term consists of two components: the first measures the change in the firm's equity value due to increased investment resulting from higher debt, while the second gauges the impact of induced investment changes on the firm's debt value. Since $\pi^i_{i\xi^i} \geq 0$, this term is positive when firm i faces cash constraints ($\mu^i_{K^i}(K) = 1$), and zero otherwise ($\mu^i_{K^i}(K) = 0$). This implies that the induced investment changes caused by additional debt reduce the conflict between debt and equity holders and increase the firm's debt value when the firm is cash-constrained in the investment sub-game.

The strategic-commitment effect evaluates the change in firm i's total value due to the impact of increased debt on competitors' equilibrium investments. This hinges on how externalities are influenced by ξ^i, denoted by $\pi^i_{j\xi^i}$, and how firm i's available cash in the investment sub-game alters competitors' investments. Initially, this effect can be positive or negative, depending on how firms compete in the product market and how investments affect demand or product cost functions, or both.

Once D^i is sufficiently large to alleviate cash constraints in the investment sub-game, $\mu(K)$ becomes independent of K^i, and both the direct and strategic-commitment effects of debt become zero.

From this point onward, we assume the following regularity condition:

Assumption 4.

$$\int_{\underline{\xi}^i}^{\bar{\xi^i}} \pi^i(\mu(K),\xi^i)dF(\xi^i) - \mu^i(K)$$

is strictly quasi-concave in K^i for all K^{-i}.[24]

Because $\mu^i(K)$ is independent of A^i, when the firm is unconstrained in the investment sub-game, the

[24] The Maximum Theorem ensures the existence of a maximum. However, a more detailed characterization of the best-response requires strict quasi-concavity.

firm will consider external financing whenever $BR^{iu}(\mu^{-i}(A^i, K^{-i})) > A^i$; otherwise, it can finance the unconstrained investment with its own cash. Hence, cash-rich firms do not request external financing.

The discussion so far leads to the following result.

Proposition 6. *Suppose Assumptions 3 and 4 hold. If $A^i \geq BR^{iu}(\mu^{-i}(A^i, K^{-i}))A^i$, then firm i's best response is $BR^i(K) = 0$, while if $A^i < BR^{iu}(\mu^{-i}(A^i, K^{-i}))$,*

i. If

$$\left(\int_{\underline{\xi}^i}^{\xi^i(D^i)} \pi^i_{i\xi^i}(\mu(K), \xi^i)(1 - F(\xi^i))d\xi^i + \sum_{j \neq = i} \mu^j_{K^i}(K) \int_{\underline{\xi}^i}^{\bar{\xi}^i} \pi^i_{j\xi^i}(\mu(K), \xi^i)(1 - F(\xi^i))d\xi^i \right) \Big|_{D^i = BR^{iu}(\mu^{-i}(K)) - A^i} \geq 0,$$

then firm i's best response is $BR^i(K) = BR^{iu}(\mu^{-i}(K^{-i})) - A^i$.

ii. If

$$\left(\int_{\underline{\xi}^i}^{\bar{\xi}^i} \pi^i_{i\xi^i}(\mu(K), \xi^i)(1 - F(\xi^i))d\xi^i + \sum_{j \neq i} \mu^j_{K^i}(K) \int_{\underline{\xi}^i}^{\bar{\xi}^i} \pi^i_{j\xi^i}(\mu(K), \xi^i)(1 - F(\xi^i))d\xi^i \right) \Big|_{D^i = 0} < 1,$$

then firm i's best response is $BR^i(K) = 0$.

iii. If neither of the two conditions above holds, then firm i's best response is the solution to

$$\left(\int_{\underline{\xi}^i}^{\bar{\xi}^i} \pi^i_{i\xi^i}(\mu(K), \xi^i)(1 - F(\xi^i))d\xi^i + \sum_{j \neq i} \mu^j_{K^i}(K) \int_{\underline{\xi}^i}^{\bar{\xi}^i} \pi^i_{j\xi^i}(\mu(K), \xi^i)(1 - F(\xi^i))d\xi^i \right) = 1,$$

and therefore $BR^i(K) \in (0, BR^{iu}(\mu^{-i}(K^{-i})) - A^i)$.

This proposition suggests that when the strategic-commitment effect of debt, evaluated at the investment sub-game equilibrium where firm i is not cash-constrained, is positive, firm i aims to borrow enough to finance the unconstrained optimal investment. This arises because when the firm is cash-constrained, not only is the marginal contribution of debt to the total firm value, net of investment cost and holding competitors' investment constant, positive, but it also exceeds the marginal return due to competitors' optimal investment responses to an increase in firm i's debt.

At $D^i = 0$, there is no conflict between debt and equity holders (as there are no debt holders), and therefore the second term inside the first parenthesis in equation (8) is zero. In this scenario, if the sum of the marginal contribution made by firm i's investment induced by positive debt does not offset the strategic-commitment effect of debt, it is optimal not to borrow money and finance the investment solely with equity, leading to cash constraints in the investment sub-game. This occurs only when the strategic-commitment effect of debt is negative. The absence of borrowing serves as a credible strategy to reduce competition intensity in the investment sub-game, as no firm can credibly commit to underinvest in the investment sub-game.

When neither of the two cases above applies, firm i aims to borrow a positive amount but less than the minimum required to be unconstrained in the investment sub-game. This decision is driven by the fact that profits decrease at the unconstrained investment sub-game equilibrium, along with the negative marginal return due to competitors' optimal investment responses to an increase in firm i's cash. Borrowing less than the required amount to be unconstrained in the investment sub-game serves as a credible strategy to reduce competition intensity in the investment sub-game, as no firm can credibly commit to underinvest in the investment sub-game.

The next result readily follows from the fact that firm i's profits are quasi-concave and continuous in K^i.

Lemma 2. *Suppose assumptions 3 and 4 hold and $BR^{iu}(\mu^{-i}(A^i, K^{-i})) > A^i$. Then, if*

$$\sum_{j \neq i} \mu^j_{K^i}(K) \int_{\underline{\xi}^i}^{\bar{\xi}^i} \pi^i_{j\xi^i}(\mu(K), \xi^i)(1 - F(\xi^i)) d\xi^i < 0,$$

when evaluated at $(D^i, A^i) = (0, BR^{iu}(\mu^{-i}(A^i, K^{-i})))$, there exists an initial cash threshold, denoted by $A^i(K^{-i})$, such that $BR^i(K) = 0$ whenever $A^i > A^i(K^{-i})$ and $BR^i(K) \in (0, BR^{iu}(\mu^{-i}(K)) - A^i]$ otherwise.

Hence, if the strategic-commitment effect is negative when evaluated at the firm's unconstrained investment level, cash-rich firms choose not to borrow money to soften competition ex-post. Firm with less cash will choose a level of external financing that will leave them cash-constrained since the benefit from investing more in the investment sub-game partially compensates for the fact that investing more increases the negative impact of competition on profits. This implies that cash-rich firms will be reluctant to hoard cash when the strategic-commitment effect is sufficiently negative since they cannot commit not to invest it in the investment sub-game.

When externalities increase with the shock ξ^i; i.e., $\pi^i_{j\xi^i}(\mu(K), \xi^i) > 0$, and investments are strategic complements, the strategic-commitment effect is positive. If investments are strategic substitutes and competitors' investments do not increase with K^i, the strategic-commitment effect is negative. In contrast, if externalities decrease with the shock ξ^i; i.e., $\pi^i_{j\xi^i} < 0$ and investments are strategic complements, the strategic-commitment effect is negative, while if investments are strategic substitutes and competitors' investments do not increase with K^i, the strategic-commitment effect is positive.

This is summarized in the next proposition.

Proposition 7. *Suppose assumptions 3 and 4 hold and $BR^{iu}(\mu^{-i}(K)) > A^i$. Then,*

i. Suppose that $\pi^i_{ij} \geq 0$ for all j. If $\pi^i_{j\xi^i} \geq 0$, firm i borrows just enough to be unconstrained in the investment sub-game, while if $\pi^i_{j\xi^i} < 0$, firm i chooses an amount of external financing so that it becomes cash-constrained in the investment sub-game.

ii. Suppose that $\pi^i_{ij} < 0$ and $\mu^j_{K^i} \leq 0$ for all j. If $\pi^i_{j\xi^i} \geq 0$, firm i chooses an amount of external financing so that it becomes cash-constrained in the investment sub-game, while if $\pi^i_{j\xi^i} < 0$, firm i borrows just enough to be unconstrained in the investment sub-game.

Firm i's strategic-commitment effect is positive when the impact of the profitability shock on the externalities that competitors have on firm i's profits has the same sign as the impact that firm i's external financing has on firm j's optimal investment. When they have the opposite sign, the strategic commitment effect is negative.

In duopolies, the requirement for externalities and complementarities to have the same sign is exactly met in Cournot, Bertrand, and Hotelling duopolies. Since cash-constrained firms do not change their investment when firm i increases its debt, only unconstrained firms' behavior matters in determining the sign of the strategic-commitment effect. This implies that the sign of the strategic-commitment effect is determined by the externalities and behavior of firms with large amounts of cash, either due to hoarding or better borrowing capacity or both. These are often large firms or firms belonging to conglomerates benefiting from cash pooling.

Because the sign of externalities depends on strategic, technological, and demand spillover effects, we need to know the details of the game played to determine the sign of externalities. However, if the demand and strategic spillover effects are negative, the externality is likely to be negative and significant for the most efficient firms, given their larger profit margins in the product-market game. Hence, it is reasonable to assume that firms' behavior regarding external financing will be driven mostly by the behavior of cash-rich or more efficient firms, or both.

It is interesting to note the following.

Corollary 2. *Suppose assumptions 3 and 4 hold. Then, a monopoly borrows the lowest amount needed to be unconstrained in the investment sub-game.*

This follows from the fact that the net present value of the investment is positive and that there is no strategic-commitment effect of debt in a monopoly.

Next, we will shed some light on how firm i's best response regarding debt changes with competitors' cash. When firm i's external financing is positive, there are two cases to consider: first, when the strategic-commitment effect is non-negative, firm i's external financing is such that it can invest $BR^{iu}(\mu^{-i}(K))$; and second, when the strategic-commitment effect is negative, the firm borrows an amount that solves the first-order condition with equality when it borrows a positive amount; otherwise it does not borrow money. In the former case, firm i borrows $BR^{iu}(\mu^{-i}(K)) - A^i$ and thereby its best response rises with K^h if and only if

$$\sum_{j\neq i} \mu^j_{K^h}(K)\left(\int_{\xi^i(D^i)}^{\bar{\xi^i}} \pi^i_{i\xi^i j}(\mu(K),\xi^i)(1-F(\xi^i))d\xi^i - \pi^i_j(\mu(K),\xi^i) f(\xi^i(D^i))\frac{\xi^i(D^i)}{\partial \mu^j}\right) > 0, \quad (9)$$

and in the latter, because $\mu^i(K) = K^i$, firm i's debt rises with firm h's debt whenever

$$\sum_{j\neq i}^{n} \mu^j_{K^i K^h}(K)\int_{\underline{\xi}^i}^{\bar{\xi^i}} \pi^i_{j\xi^i}(\mu(K),\xi^i)(1-F(\xi^i))d\xi^i +$$
$$\sum_{l=1}^{n} \mu^l_{K^h}(K)\sum_{j=1}^{n}\mu^j_{K^i}(K)\int_{\underline{\xi}^i}^{\bar{\xi^i}} \pi^i_{j\xi^i l}(\mu(K),\xi^i)(1-F(\xi^i))d\xi^i > 0.$$

If firm i borrows an amount such that the firm is unconstrained in the investment sub-game, then as firm h borrows more, firm i borrows more when either investment are strategic complements or they are strategic substitutes and its competitors' optimal investment fall with their competitors' borrowing. Otherwise, it decreases with competitors' borrowing.

When firm i borrows a positive amount that leaves it cash-constrained in the investment sub-game, the impact of competitors' cash on firm i's best response consists on two different terms: The first one is positive when investments are supermodular in K and $\pi^i_{j\xi^i} \geq 0$ or submodular and $\pi^i_{j\xi^i} < 0$; otherwise, it is negative. The second term regards how the marginal return to the shock changes with the change in each possible pair of investments due to changes in their corresponding cash levels. This term depends on how the shock ξ^i affect the degree of complementarity/substitutability of investments. Thus, the sign of both term is a-priori ambiguous.

Within this general setting, we can neither devise the conditions on the primitives of the product-market game such that $\pi^i_{j\xi^i l}(\mu(K),\xi^i)$ is either positive or negative nor we can provide a more detailed characterization of the equilibrium. The existence of an equilibrium follows from quasi-concavity and Theorem 1 in

Rosen (1965), uniqueness, however, is not guaranteed.[25] Hence, we have the following result.

Proposition 8. *Suppose assumptions 3 and 4 hold. Then, there is a sub-game perfect equilibrium given by* $(\mu(A), D(A))$. *Firms choose to finance their investments with pure debt and the initial cash.*

In equilibrium, firms' financing and investment behavior are strategically intertwined in the following way: when the strategic-commitment effect is positive, firms use external financing in the form of a standard debt contract. They borrow enough to avoid being cash-constrained in the investment sub-game, and they invest the totality of their cash in that phase. Conversely, when the strategic-commitment effect is negative, firms may choose to abstain from borrowing or borrow a positive amount that leaves them cash-constrained in the investment sub-game. They utilize a standard debt contract for borrowing, and in the investment sub-game, firms invest the entirety of the available cash but despite this they underinvest. This decision is due to that firms cannot commit ex-ante not to invest their cash whenever it is lower than the unconstrained optimal investment.

These results have several interesting implications. Firstly, the optimal financial contract is a standard debt contract.

Secondly, there is a complex relationship between any firm's borrowing capacity and its competitors' borrowing capacity that cannot be devised analytically and therefore it needs to be studied empirically by considering complementarities and externalities.

Thirdly, firms' external financing strategies are determined by the sign of the strategic-commitment effect. This depends on how the profitability shock affects externalities, competitors' optimal investment responses to the corresponding firm's debt, the type of competition and its intensity in the product-market game, the demand response to the shock. the technology spillovers, and initial cash holdings.

When the strategic-commitment effect is non-negative, firms choose external financing so that they are not cash-constrained in the investment sub-game, while when the strategic-commitment effect is negative, firms choose external financing so as to be cash-constrained in the investment sub-game. Thus, empirical models that study the impact of cash constraints on investment should carefully define the appropriate market, identify externalities, as done, for instance, in Bloom et al. (2013), and somehow take into account competitors' investment behavior in the investment sub-game.

Fourthly, because being cash-constrained in the investment sub-game is a strategic decision, the best

[25] Rosen's (1965) Theorem 2 shows that a sufficient condition for uniqueness is that $\sum_{i=1}^{n} r^i\big(\pi^i(\mu(K)) - c^i(\mu^i(K))\big)$ is diagonally strictly concave for a given non-negative vector $r \in E^n$, where E^n is the Euclidean space in n, since the constraint set is convex, closed and bounded. If the Jacobian of the equilibrium conditions and its transpose are both B_0-matrices, then its sum is negative definite (see, Christensen (2018)) and, thereby, Rosen's (1965) diagonally strict concavity property holds. Hence, the equilibrium will be unique.

policy to increase investment is a cash-in-hand policy or a tax-break policy since firms cannot commit ex-post to not invest the available cash, while softening credit constraints with cheap credit lines does not induce more investment when the strategic-commitment effect is negative.

6 Conclusions

This paper demonstrates that under certain conditions regarding marginal returns the optimal way to finance investment in oligopolistic markets is through a standard debt contract. Firms choose the debt level ex-ante with the goal of softening competition in the investment sub-game. Despite the option to borrow more, firms may choose to be cash-constrained when the time to invest arrives. This decision arises because they cannot commit to not investing optimally, deliberately constraining their cash to mitigate competition intensity. Consequently, being cash-constrained in the investment sub-game is an optimal strategic response to temper competition at that stage, challenging the conclusion that investment is limited only due to firms' lack of better access to external financing.

The key determinants of when to be cash-constrained are externalities (strategic and technological spillovers), investment complementarity, and initial cash. These factors determine the sign and magnitude of the strategic-commitment effect of debt. When this effect is non-negative, firms borrow as much as needed to avoid being cash-constrained in the investment sub-game. Conversely, when the strategic-commitment effect of debt is negative, firms borrow an amount that leaves them cash-constrained during the investment sub-game.

The primary market failures justifying government intervention in investment are positive spillovers and limited access to external financing. The rationale for the former is rooted in the idea that investing firms do not fully internalize spillover effects, while the latter is based on the intangible, uncertain, and non-contractible nature of innovations, resulting in limited borrowing capacity. Empirical evidence supports both the existence of uninternalized positive spillovers (where the private rate of return falls short of the social rate of return) and a positive relationship between investment and external financing.

In cases where the strategic-commitment effect of debt is negative, facilitating access to credit will not be helpful. However, other types of incentives, such as a cash-in-hand policy or tax breaks, can be effective. A cash-in-hand policy is effective because firms cannot commit to not investing the available cash in the investment sub-game. Both the firm receiving the cash and industry-wide investment when our sufficient conditions holds increase with a positive cash shocks. Tax breaks work by increasing the marginal return on investment. Both policies can improve efficiency, even when spillovers are negative, as firms choose to be cash-constrained ex-post and may invest less than the socially efficient amount. Empirical evidence also indicates that tax breaks lead to increased investment. The literature concludes that the elasticity of R&D with respect to the tax rate is equal to minus one and recent evidence points to a causal positive impact of R&D tax breaks on corporate investment.

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A Proofs

Proof of Proposition 1. Observe that the marginal return to the investment μ^{-i} under contract R^{i0} is lower than under contract R^{i1} iff

$$\int_{\xi^i \in \Xi} (1 - R_i^{i0}(\pi^i(\mu, \xi^i)))\pi_i^i(\mu, \xi^i) dF(\xi^i) \leq \int_{\xi^i \in \Xi} (1 - R_i^{i1}(\pi^i(\mu, \xi^i)))\pi_i^i(\mu, \xi^i) dF(\xi^i),$$

iff

$$\begin{aligned}
&\int_{\xi^i \in \Xi} \left(R_i^{i1}(\pi^i(\mu, \xi^i)) - R_i^{i0}(\pi^i(\mu, \xi^i))\right)\pi_i^i(\mu, \xi^i) dF(\xi^i) \\
=&\int_{\underline{\xi}^i}^{\hat{\xi}^i} \left(R_i^{i1}(\pi^i(\mu, \xi^i)) - R_i^{i0}(\pi^i(\mu, \xi^i))\right)\pi_i^i(\mu, \xi^i) dF(\xi^i) + \int_{\hat{\xi}^i}^{\bar{\xi}^i} \left(R_i^{i1}(\pi^i(\mu, \xi^i)) - R_i^{i0}(\pi^i(\mu, \xi^i))\right)\pi_i^i(\mu, \xi^i) dF(\xi^i) \\
\leq&\pi_i^i(\mu, \hat{\xi}^i) \int_{\xi^i \in \Xi} \left(R_i^{i1}(\pi^i(\mu, \xi^i)) - R_i^{i0}(\pi^i(\mu, \xi^i))\right) dF(\xi^i) \\
\leq&0
\end{aligned}$$

where the first inequality follows from the fact that $R_i^{i1}(\pi^i(\mu, \xi^i)) - R_i^{i0}(\pi^i(\mu, \xi^i)) \leq 0$ for all $\xi^i \leq \hat{\xi}^i$ and $R_i^{i1}(\pi^i(\mu, \xi^i)) - R_i^{i0}(\pi^i(\mu, \xi^i)) > 0$ for all $\xi^i > \hat{\xi}^i$ and $\pi_{i\xi^i}^i \geq 0$, and the last inequality follows from the fact that

$$\int_{\xi^i \in \Xi} R^{i0}(\pi^i(\mu, \xi^i))) dF(\xi^i) = \int_{\xi^i \in \Xi} R^{i0}(\pi^i(\mu, \xi^i))) dF(\xi^i),$$

and $R^{i0}(\pi(\mu, \underline{\xi}^i)) = R^{i1}(\pi(\mu, \underline{\xi}^i))$.

Next, observe that by optimality $\Pi^i(\mu^{i0}, \mu^{-i}, R^{i1}) - \mu^{i0} \leq \Pi^i(\mu^{i1}, \mu^{-i}, R^{i1}) - \mu^{i1}$. Because for all μ, $\Pi_i^i(\mu^i, \mu^{-i}, R^{i0}) - 1 \leq \Pi_i^i(\mu^i, \mu^{-i}, R^{i1}) - 1$, $\Pi^i(\mu^{i0}, \mu^{-i}, R^{i0}) - \mu^{i0} \leq \Pi^i(\mu^{i0}, \mu^{-i}, R^{i1}) - \mu^{i0}$. Hence, $\Pi^i(\mu^{i0}, \mu^{-i}, R^{i0}) - \mu^{i0} \leq \Pi^i(\mu^{i1}, \mu^{-i}, R^{i1}) - \mu^{i1}$.

□

Proof of Proposition 4. Let's assume that the first k firms are cash-constrained, then from the equilibrium conditions we deduce the following

$$\begin{pmatrix} -1 & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & -1 & 0 & \dots & \dots & \dots & 0 \\ \vdots & \dots & \ddots & \dots & \dots & \dots & \vdots \\ 0 & \ddots & \dots & -1 & 0 & \dots & 0 \\ \Pi^{k+1}_{k+1,1} & \ddots & \dots & \dots & \Pi^{k+1}_{k+1,k+1} & \dots & \Pi^{k+1}_{k+1,n} \\ \vdots & \dots & \dots & \dots & \dots & \ddots & \dots \\ \Pi^{n}_{n,1} & \dots & \dots & \dots & \dots & \dots & \Pi^{n}_{n,n} \end{pmatrix} \times \begin{pmatrix} \mu^1_{K^1} \\ \vdots \\ \vdots \\ \mu^k_{K^1} \\ \vdots \\ \vdots \\ \mu^n_{K^1} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{A1}$$

Because $-\Pi$ is a B_0 matrix, the negative of the first matrix, denoted by $-\Pi^c$, is also a B_0 matrix since for the first k rows, the row sum is lower than or equal to n times the lowest between zero and the off-diagonal row element since each of them is zero and $-1 \leq 0$ for all $i \in \{1, \dots, k\}$. The same holds for row $k+1$ to n due to the fact that $-\Pi$ is a B_0 matrix. Let Π^c_{ij} be the co-factor i and j. Let's denote the unknown vector by μ_{K^1} and the last vector by b. Then, $\Pi^c \mu_{K^1} = b$. Multiplying both side by -1, one gets that $\mu_{K^1} = (-\Pi^c)^{-1}(-b)$ and $\mu^i_{K^1} = \frac{1}{\det(-\Pi^c)} \sum_{j=1}^n (-\Pi^c_{ij})(-b_j) = \frac{(-\Pi^c_{i1})}{\det(-\Pi^c)}$. It follows then

$$\begin{aligned} \sum_{i=1}^n \mu^i_{K^1} =& \frac{1}{\det(-\Pi^c)} \sum_{i=1}^n \sum_{j=1}^n (-b_j)(-\Pi^c_{ij}) \\ =& \frac{1}{\det(-\Pi^c)} \sum_{j=1}^n \Big(\sum_{i=1}^n (-\Pi^c_{ij}) \Big)(-b_j) = \frac{1}{\det(-\Pi^c)} \Big(\sum_{i=1}^n (-\Pi^c_{i1}) \Big) \end{aligned}$$

Hence, this is non-negative if and only if $\sum_{i=1}^n (-\Pi^c_{i1}) \geq 0$, which is the case when $-\Pi^c$ is a B_0-matrix. This follows from substituting the first-row in $-\Pi^c$ by a row of -1 and then the determinant of this matrix is $\sum_{i=1}^n (-\Pi^c_{i1})$, which is positive because the new matrix with -1s in row 1 is a B_0 matrix since the row sum of -1s is lower than or equal to n times the lowest between zero and the smallest off-diagonal row element, which is -1.

□

Proof of Proposition 5. Observe that $K^i \equiv A^i + \int_{\xi^i \in \Xi} R^i_i(\pi^i(\mu(K), \xi^i))\pi^i_{\xi^i}(\mu(K), \xi^i)(1 - F(\xi^i))d\xi^i + R^i(\pi^i(\mu(K), \underline{\xi}^i)$. We derive the financing contract that maximize profits when implements $\mu(K)$ for any given K. Then, we will look for the optimal K given the best contract for any K. Then, integration-by-parts, firm i's optimization problem is as follows

$$\max_{R^i \in \mathcal{R}} \int_{\xi^i \in \Xi} \Big(\pi^i_{\xi^i}(\mu(K),\xi^i)(1 - R^i_i(\pi^i(\mu(K),\xi^i)))(1-F(\xi^i))d\xi^i + \pi^i(\mu(K),\underline{\xi}^i) - R^i(\pi^i(\mu(K),\underline{\xi}^i)) \tag{A2}$$

subject to

$$\int_{\xi^i \in \Xi} R^i_i(\pi^i(\mu(K),\xi^i))\pi^i_{\xi^i}(\mu(K),\xi^i)(1-F(\xi^i))d\xi^i + R^i(\pi^i(\mu(K),\underline{\xi}^i) = D^i,$$

$$\int_{\xi^i \in \Xi} \big(1 - R^i_i(\pi^i(\mu(K),\xi^i))\big)\pi^i_i(\mu(K),\xi^i)dF(\xi^i) = 1 + \lambda^i,$$

$$0 \leq R^i(\pi^i(\mu(K),\xi^i)) \leq \pi^i(\mu(K),\xi^i),\ \forall\, \xi^i,$$

$$R^i_i(\pi^i(\mu(K),\xi^i))\pi^i_{\xi^i}(\mu(K),\xi^i) \geq 0,\ \text{a.e.}.$$

Lets's define the control $c^i(\xi^i) \equiv R^i_i(\pi^i(\mu(K),\xi^i))\pi^i_{\xi^i}(\mu(K),\xi^i)$ and focus on the case in which $D^i > \pi^i(\mu(K),\underline{\xi}^i)$. We will impose the initial condition $R^i(\pi^i(\mu(K),\underline{\xi}^i)) = \underline{R} \leq \pi^i(\mu(K),\underline{\xi}^i)$ Then, the Lagrangean is given by

$$\begin{aligned}\mathcal{L}(\xi^i, R^i, c^i, \dot{\Gamma}, \dot{\Phi}) =& \mathcal{H}(\xi^i, R^i, c^i, \dot{\Gamma}, \dot{\Phi}) + \delta(\xi^i)R^i(\pi^i(\mu(K),\xi^i)) + \\ & \nu(\xi^i)(\pi^i(\mu(K),\xi^i) - R^i(\pi^i(\mu(K),\xi^i))) + \gamma(\xi^i)c^i(\xi^i)\end{aligned} \tag{A3}$$

where

$$\begin{aligned}\mathcal{H}(\xi^i, R^i, c^i, \dot{\Gamma}, \dot{\Phi}) \equiv& (\pi^i_{\xi^i}(\mu(K),\xi^i) - c^i(\xi^i))(1-F(\xi^i)) + \alpha(\xi^i)c^i(\xi^i) + \rho(\xi^i)c^i(\xi^i)(1-F(\xi^i)) + \\ & \beta(\xi^i)\big(\pi^i_{\xi^i}(\mu(K),\xi^i) - c^i(\xi^i)\big)\frac{\pi^i_i(\mu(K),\xi^i)}{\pi^i_{\xi^i}(\mu(K),\xi^i)}f(\xi^i),\end{aligned}$$

$$\Gamma(\xi^i) = \int_{\underline{\xi}^i}^{\xi^i} \big(\pi^i_{\xi^i}(\mu(K),x) - c^i(x)\big)\frac{\pi^i_i(\mu(K),x)}{\pi^i_{\xi^i}(\mu(K),x)}dF(x),$$

$$\dot{\Gamma}(\xi^i) = \big(\pi^i_{\xi^i}(\mu(K),\xi^i) - c^i(\xi^i)\big)\frac{\pi^i_i(\mu(K),\xi^i)}{\pi^i_{\xi^i}(\mu(K),\xi^i)}f(\xi^i),$$

with $\Gamma(\underline{\xi}^i) = 0$ and $\Gamma(\bar{\xi^i}) = 1 + \lambda$ for all $\xi^i \in \Xi \setminus \bar{\xi^i}$,

$$\Phi(\xi^i) = \int_{\underline{\xi}^i}^{\xi^i} c^i(x)(1-F(x))dx,$$

and

$$\dot{\Phi}(\xi^i) = c^i(\xi^i)(1 - F(\xi^i)),$$

with $\Phi(\underline{\xi}^i) = 0$ and $\Phi(\bar{\xi^i}) = D^i$ for all $\xi^i \in \Xi \setminus \bar{\xi^i}$.

The optimality conditions are

$$\begin{aligned}
&\dot{\alpha}(\xi^i) = -\frac{\partial \mathcal{L}}{\partial R^i} = -\delta(\xi^i) + \nu(\xi^i), \\
&\dot{\beta}(\xi^i) = -\frac{\partial \mathcal{L}}{\partial \Gamma} = 0, \\
&\dot{\rho}(\xi^i) = -\frac{\partial \mathcal{L}}{\partial \Phi} = 0, \\
&\frac{\partial \mathcal{L}}{\partial c^i} = 0 \Longrightarrow -(1 - F(\xi^i)) - \beta(\xi^i)\frac{\pi^i_i(\mu(K), \xi^i)}{\pi^i_{\xi^i}(\mu(K), \xi^i)} f(\xi^i) + \rho(\xi^i)(1 - F(\xi^i)) + \alpha(\xi^i) + \gamma(\xi^i) = 0, \\
&\Gamma(\bar{\xi^i}) = 1 + \lambda \text{ and } \Gamma(\underline{\xi}^i) = 0, \\
&\Phi(\underline{\xi}^i) = 0 \text{ and } \Phi(\bar{\xi^i}) = D^i - R^i(\pi^i(\mu(K), \underline{\xi}^i), \\
&c^i(\xi^i) \geq 0,\ \gamma(\xi^i) \geq 0,\ \text{and } \gamma(\xi^i)c^i(\xi^i) = 0, \\
&R^i(\pi^i(\mu(K), \xi^i)) \geq 0,\ \delta(\xi^i) \geq 0,\ \text{and } \delta(\xi^i)R^i(\pi^i(\mu(K), \xi^i)) = 0, \\
&\pi^i(\mu(K), \xi^i) - R^i(\pi^i(\mu(K), \xi^i)) \geq 0,\ \nu(\xi^i) \geq 0,\ \text{and } \nu(\xi^i)(\pi^i(\mu(K), \xi^i) - R^i(\pi^i(\mu(K), \xi^i))) = 0.
\end{aligned}$$

The co-state variable $\alpha(\xi^i)$ must be piece-wise continuous differentiable and have jumps only at junction points. The co-states $\beta(\xi^i)$ and $\rho(\xi^i)$ are constants. Let's assume that they are β and ρ respectively.

It readily follows from the second optimality condition that $\beta(\xi^i)$ is constant and from the first that $\alpha(\xi^i) = \alpha(\underline{\xi}^i) - \int_{\underline{\xi}^i}^{\xi^i} (\delta(x) - \nu(x))dx$ respectively. Because the end point is free, $\alpha(\bar{\xi^i}) = 0$ and therefore $\alpha(\underline{\xi}^i) = \int_{\underline{\xi}^i}^{\bar{\xi^i}} (\delta(x) - \nu(x))dx$. It follows from this that

$$\alpha(\xi^i) = \int_{\xi^i}^{\bar{\xi^i}} (\delta(x) - \nu(x))dx.$$

Substituting this into the first-order condition for the control variable, we deduce that

$$\gamma(\xi^i) = \Big(\beta\frac{\pi^i_i(\mu(K), \xi^i)}{\pi^i_{\xi^i}(\mu(K), \xi^i)} H(\xi^i) + 1 - \rho\Big)(1 - F(\xi^i)) - \int_{\xi^i}^{\bar{\xi^i}} (\delta(x) - \nu(x))dx \geq 0,$$

where $H(\xi^i) \equiv \frac{f(\xi^i)}{1 - F(\xi^i)}$.

Because $\gamma(\xi^i) \geq 0$ for all $\xi^i \in \Xi \setminus \bar{\xi^i}$, $\delta(\xi^i) = 0$ and $\nu(\xi^i) > 0$ whenever

$$\Big(\beta \frac{\pi_i^i(\mu(K), \xi^i)}{\pi_{\xi^i}^i(\mu(K), \xi^i)} H(\xi^i) + 1 - \rho\Big)(1 - F(\xi^i)) < 0,$$

while $\delta(\xi^i)$ could be strictly positive whenever the opposite holds.

Evaluating the first-order condition for $c^i(\xi^i)$ at $\xi^i = \bar{\xi^i}$, it is easy to check that $\rho = \beta \frac{\pi_i^i(\mu(K),\bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K),\bar{\xi^i})} H(\bar{\xi^i})$. Multiplying the first-order condition for $c^i(\xi^i)$ by $c^i(\xi^i)$, integrating over Ξ, using the fact that $\gamma(\xi^i)c^i(\xi^i) = 0$, $\Gamma(\bar{\xi^i}) = 1 + \lambda$, $\Phi(\bar{\xi^i}) = D^i - R^i(\pi^i(\mu(K), \underline{\xi}^i)$, it is easy to see that the optimal β, denoted by $\beta(D^i, \lambda)$, is equal to

$$\beta = \frac{D^i - R^i(\pi^i(\mu(K), \underline{\xi}^i) - \int_{\underline{\xi}^i}^{\bar{\xi^i}} \int_{\xi^i}^{\bar{\xi^i}} (\delta(x) - \nu(x)) dx c^i(\xi^i) d\xi^i}{\frac{\pi_i^i(\mu(K),\bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K),\bar{\xi^i})} H(\bar{\xi^i})(D^i - R^i(\pi^i(\mu(K), \underline{\xi}^i)) + 1 + \lambda - \int_{\underline{\xi}^i}^{\bar{\xi^i}} \pi_i^i(\mu(K), \xi^i) dF(\xi^i)}.$$

Observe that $\int_{\underline{\xi}^i}^{\bar{\xi^i}} \int_{\xi^i}^{\bar{\xi^i}} (\delta(x) - \nu(x)) dx c^i(\xi^i) d\xi^i$ takes the value zero for all ξ^i such that $\delta(\xi^i) > 0$ since $R^i(\pi^i(\mu(K), \xi^i)) = 0$ and for those ξ^i such that $\delta(\xi^i) = \nu(\xi^i) = 0$. Hence, this term is non-zero only for those ξ^i for which $\nu(\xi^i) > 0$. When this is the case, $c^i(\xi^i) = \pi_{\xi^i}^i(\mu(K), \xi^i) > 0$. Hence, $\int_{\underline{\xi}^i}^{\bar{\xi^i}} \int_{\xi^i}^{\bar{\xi^i}} (\delta(x) - \nu(x)) dx c^i(\xi^i) d\xi^i \leq 0$.

Observe that denominator can be written as follows

$$\begin{aligned}
&\int_{\underline{\xi}^i}^{\bar{\xi^i}} c^i(\xi^i)(1 - F(\xi^i)) \frac{\pi_i^i(\mu(K), \bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K), \bar{\xi^i})} H(\bar{\xi^i}) d\xi^i - \int_{\underline{\xi}^i}^{\bar{\xi^i}} c^i(\xi^i) \frac{\pi_i^i(\mu(K), \xi^i)}{\pi_{\xi^i}^i(\mu(K), \xi^i)} f(\xi^i) d\xi^i \\
> &\int_{\underline{\xi}^i}^{\bar{\xi^i}} c^i(\xi^i)(1 - F(\xi^i)) \frac{\pi_i^i(\mu(K), \xi^i)}{\pi_{\xi^i}^i(\mu(K), \xi^i)} H(\xi^i) d\xi^i - \int_{\underline{\xi}^i}^{\bar{\xi^i}} c^i(\xi^i) \frac{\pi_i^i(\mu(K), \xi^i)}{\pi_{\xi^i}^i(\mu(K), \xi^i)} f(\xi^i) d\xi^i \\
= &0,
\end{aligned}$$

where the inequality follows from the fact that It readily follows from the discussion up to here that $\frac{\pi_i^i(\mu(K),\xi^i)}{\pi_{\xi^i}^i(\mu(K),\xi^i)} H(\xi^i)$ rises with ξ^i. Hence, the denominator is positive. This together with the fact that numerator is positive implies that $\beta(D^i, \lambda) > 0$.

Next, observe that

$$\gamma(\xi^i) = \Big(\beta(D^i, \lambda)\Big(\frac{\pi_i^i(\mu(K), \xi^i)}{\pi_{\xi^i}^i(\mu(K), \xi^i)} H(\xi^i) - \frac{\pi_i^i(\mu(K), \bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K), \bar{\xi^i})} H(\bar{\xi^i})\Big) + 1\Big)(1 - F(\xi^i)) - \int_{\xi^i}^{\bar{\xi^i}} (\delta(x) - \nu(x)) dx \geq 0$$

Because of Assumption 3 and $H(\xi^i)$ is non-decreasing, the term multiplying $\beta(D^i, \lambda)$ is negative for all

$\xi^i < \bar{\xi^i}$ and since $\beta(D^i, \lambda) > 0$, the first term inside the parenthesis is negative.

Let's guess the following solution. There exists a threshold $\hat{\xi^i} \in (\underline{\xi}^i, \bar{\xi^i})$ such that the optimal solution $\delta(\xi^i) = \nu(\xi^i) = 0$ for all $\xi^i \geq \hat{\xi^i}$ and that $\nu(\xi^i) > 0$ and $\delta(\xi^i) = 0$ for all $\xi^i \leq \hat{\xi^i}$. Then, $\int_{\xi^i}^{\bar{\xi^i}} (\delta(x) - \nu(x))dx = -\int_{\xi^i}^{\hat{\xi^i}} \nu(x)dx$. Hence, this is negative for all $\xi^i < \hat{\xi^i}$ and zero for all $\xi^i \geq \hat{\xi^i}$.

Let $\hat{\xi^i}$ be solution to $\beta(D^i, \lambda)\Big(\frac{\pi_i^i(\mu(K),\xi^i)}{\pi_{\xi^i}^i(\mu(K),\xi^i)}H(\xi^i) - \frac{\pi_i^i(\mu(K),\bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K),\bar{\xi^i})}H(\bar{\xi^i})\Big) = -1$. The existence and uniqueness of $\hat{\xi^i}$ follow from the monotonicity of the LHS, the fact that at $\xi^i = \bar{\xi^i}$ the LHS is 0 at $\xi^i = \underline{\xi}^i$, the LHS is lower than -1, and the Intermediate Value theorem.

We need to prove the last condition, which entails the following

$$\beta(D^i, \lambda)\Big(\frac{\pi_i^i(\mu(K), \underline{\xi}^i)}{\pi_{\xi^i}^i(\mu(K), \underline{\xi}^i)}H(\underline{\xi}^i) - \frac{\pi_i^i(\mu(K), \bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K), \bar{\xi^i})}H(\bar{\xi^i})\Big) < -1.$$

After some steps of simple algebra, it can be shown that this is equivalent to

$$\int_{\underline{\xi}^i}^{\bar{\xi^i}} c^i(\xi^i)(1 - F(\xi^i))\Big(\frac{\pi_i^i(\mu(K), \underline{\xi}^i)}{\pi_{\xi^i}^i(\mu(K), \underline{\xi}^i)}H(\underline{\xi}^i)d\xi^i - \frac{\pi_i^i(\mu(K), \xi^i)}{\pi_{\xi^i}^i(\mu(K), \xi^i)}H(\xi^i)\Big)d\xi^i -$$
$$\Big(\frac{\pi_i^i(\mu(K), \underline{\xi}^i)}{\pi_{\xi^i}^i(\mu(K), \underline{\xi}^i)}H(\underline{\xi}^i) - \frac{\pi_i^i(\mu(K), \bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K), \bar{\xi^i})}H(\bar{\xi^i})\Big)\int_{\underline{\xi}^i}^{\bar{\xi^i}}\int_{\xi^i}^{\bar{\xi^i}}(\delta(x) - \nu(x))dxc^i(\xi^i)d\xi^i < 0.$$

The inequality holds always due to the fact that $\frac{\pi_i^i(\mu(K),\bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K),\bar{\xi^i})}H(\bar{\xi^i})$ rises with ξ^i and $\int_{\underline{\xi}^i}^{\bar{\xi^i}}\int_{\xi^i}^{\bar{\xi^i}}(\delta(x) - \nu(x))dxc^i(\xi^i)d\xi^i \leq 0$ and therefore the first term is negative while the second is positive.

Because $\frac{\pi_i^i(\mu(K),\bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K),\bar{\xi^i})}H(\bar{\xi^i})$ rises with ξ^i rises with ξ^i and $\gamma(\hat{\xi^i}) = 0$, $\gamma(\xi^i) > 0$ for all $\xi^i > \hat{\xi^i}$ and by choosing

$$\int_{\xi^i}^{\hat{\xi^i}} \nu(x)dx = -\Big(\beta(D^i, \lambda)\Big(\frac{\pi_i^i(\mu(K), \xi^i)}{\pi_{\xi^i}^i(\mu(K), \xi^i)}H(\xi^i) - \frac{\pi_i^i(\mu(K), \bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K), \bar{\xi^i})}H(\bar{\xi^i})\Big) + 1\Big)(1 - F(\xi^i)) > 0$$

for all $\xi^i < \hat{\xi^i}$, $\gamma(\xi^i) = 0$ for all $\xi^i < \hat{\xi^i}$. This implies that $\gamma(\xi^i)$ for all $\xi^i > \hat{\xi^i}$ and therefore $c^i(\xi^i) = 0$ for all $\xi^i > \hat{\xi^i}$ and $\nu(x) > 0$ and $\gamma(\xi^i) = 0$ for all $\xi^i < \hat{\xi^i}$. Thus, the optimal contract is a standard debt contract

$$R^i(\pi^i(\mu(K), \xi^i)) = \begin{cases} \pi^i(\mu(K), \xi^i) & \text{if } \xi^i \in (\underline{\xi}^i, \hat{\xi^i}], \\ \pi^i(\mu(K), \hat{\xi^i}) & \text{if } \xi^i \in [\hat{\xi^i}, \bar{\xi^i}], \end{cases} \tag{A4}$$

Let's denote $\hat{\xi^i}$ as $\xi^i(D^i)$, where $\xi^i(D^i)$ is the unique solution to

$$\beta(D^i, \lambda)\left(\frac{\pi_i^i(\mu(K), \xi^i)}{\pi_{\xi^i}^i(\mu(K), \xi^i)}H(\xi^i) - \frac{\pi_i^i(\mu(K), \bar{\xi^i})}{\pi_{\xi^i}^i(\mu(K), \bar{\xi^i})}H(\bar{\xi^i})\right) = -1$$

.

□